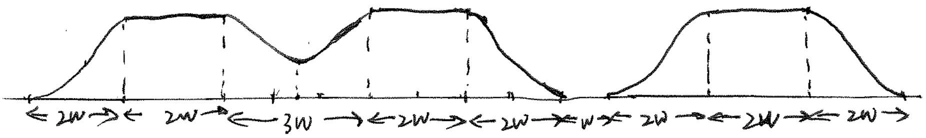
Medical Imaging, Spring 2013, Midterm Exam Solution.

1. (a) $FWHM = \frac{W}{2} \times 2 = W$ (mm)

resolution $= \frac{1}{FWHM} = \frac{1}{W}$ lines/mm

(b)



The first and the second bar cannot be told apart

The second and third bar can be.

2. (a)

$$FPP = \frac{\int_t^{\infty} P_N(x)\,dx}{\int_{-\infty}^{\infty} P_N(x)\,dx} = \int_t^{\infty} P_N(x)\,dx = \int_t^{\infty} \frac{a}{2} e^{-a(x-\mu_n)}$$

$$= -\frac{1}{a}\cdot\frac{a}{2} e^{-a(x-\mu_n)}\Big|_t^{\infty} = \frac{1}{2} e^{-a(t-\mu_n)} \quad (\text{assuming } t \geq \mu_n)$$

(b)

$$FNP = \frac{\int_{-\infty}^{t} P_d(x)\,dx}{\int_{-\infty}^{\infty} P_d(x)\,dx} = \int_{-\infty}^{t} P_d(x)\,dx = \int_{-\infty}^{t} \frac{b}{2} e^{b(x-\mu_d)}\,dx$$

$$= \frac{1}{b}\cdot\frac{b}{2} e^{b(x-\mu_n)}\Big|_{-\infty}^{t} = \frac{1}{2} e^{b(t-\mu_n)} \quad (\text{assuming } t \leq \mu_d)$$

(c) When increasing t, FPP will decrease, FNP will increase.

(d) In general, FNP should be very small if not zero in clinic

d should be larger than c to give FNP more weight.

(e) $f(t) = c\cdot FPP + d\cdot FNP = \frac{c}{2} e^{-a(t-\mu_n)} + \frac{d}{2} e^{b(t-\mu_d)}$

$$\frac{df(t)}{dt} = -\frac{ac}{2} e^{-a(t-\mu_n)} + \frac{bd}{2} e^{b(t-\mu_d)} = 0$$

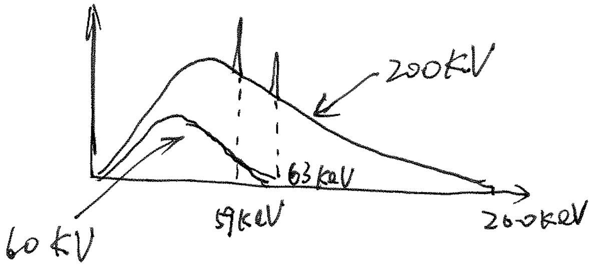
$$ac\,e^{-a(t-\mu_n)} = bd\,e^{b(t-\mu_d)} \Rightarrow t = \frac{a\mu_n + b\mu_d + \ln\frac{ac}{bc}}{a+b}$$

3. (a) The heat up of the filament causes discharge of electrons within cathode. After the tube voltage is being applied, electrons are accelerated toward the anode. These energetic electrons bombard the target and generate X-ray.

When the electrons collide~~s~~ with a K-shell ~~ele~~ electron, the atom is excited or ~~ionized~~ ionized, leaving a hole in the shell. As the atom returns to its ground state, the K-shell hole is filled by a higher shell electron. The loss of energy creates characteristic X-ray. Sometimes, the ~~ele~~ electron may bend around nucleus, caused by the positive charge of nuclei and decelerate. The loss of energy leads to Bremsstrahlung X-ray.

(b)

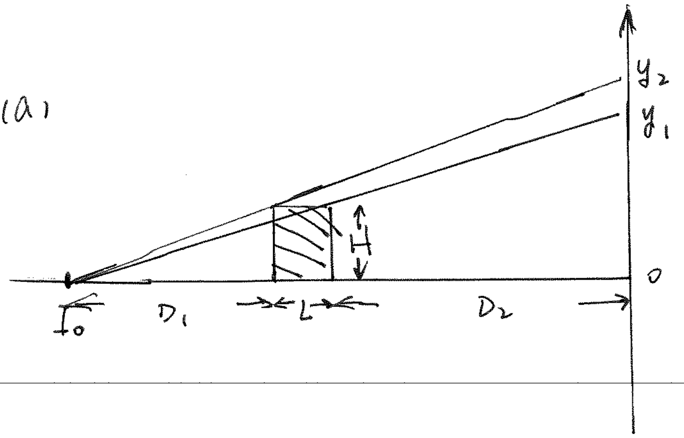
(c)



With 60kV voltage, the K-shell electrons cannot be ionized, and therefore there is no characteristic X-ray.

(d) Barium is used because it has a large attenuation coefficient. It has K-shell ~~ele~~ electrons whose binding energy falls within the diagnostic X-ray energy range. This K-edge absorption effect significantly increases the attenuation coefficient of the material in X-ray energies slightly higher than the K-shell energy.

4. (a)



when $y<0$, or $y \geq y_2$ $y_2 = (1+\frac{D_2+L}{D_1})H$

$$I(y) = I_0 \frac{\cos^3\theta}{4\pi d^2}, \text{ where } \cos\theta = \frac{D_1+L+D_2}{\sqrt{(D_1+L+D_2)^2+y^2}}. \quad d = D_1+L+D_2$$

$$= I_0 \frac{1}{4\pi(D_1+L+D_2)^2}\left(\frac{D_1+L+D_2}{\sqrt{(D_1+L+D_2)^2+y^2}}\right)^3$$

when $0 \leq y \leq y_1$, $y_1 = (1+\frac{D_2}{D_1+L})H$

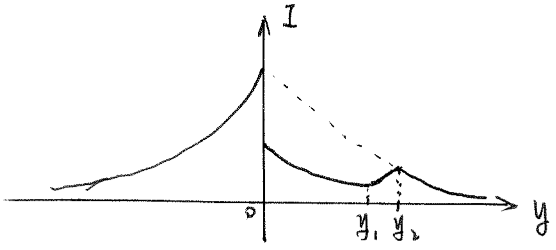
$$I(y) = I_0 \frac{1}{4\pi d^2}\cos^3\theta \cdot e^{-\mu\frac{L}{\cos\theta}} = I_0 \frac{1}{4\pi(D_1+L+D_2)^2}\left(\frac{D_1+L+D_2}{\sqrt{(D_1+L+D_2)^2+y^2}}\right)^3 \cdot e^{-\mu\frac{L\sqrt{(D_1+L+D_2)^2+y^2}}{D_1+L+D_2}}$$

when $y_1 \leq y \leq y_2$,

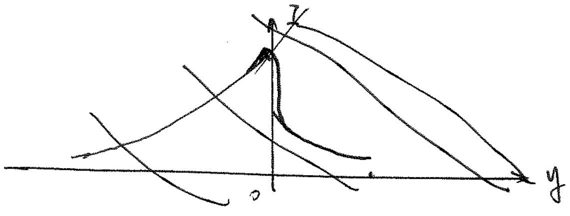
$$I(y) = I_0 \frac{1}{4\pi(D_1+L+D_2)^2} \cdot \left(\frac{D_1+L+D_2}{\sqrt{(D_1+L+D_2)^2+y^2}}\right)^3 \cdot e^{-\mu l}$$

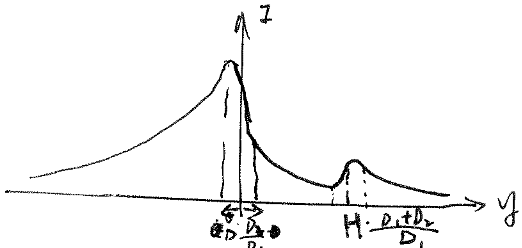
l is the length of path the x-ray passed through the object.

$$l = \frac{H}{\sin\theta} - \frac{D_1}{\cos\theta}, \quad \text{where } \sin\theta = \frac{y}{\sqrt{(D_1+L+D_2)^2+y^2}}$$



(b)





Problem 5

T2 relaxation causes a decay of the transverse component of magnetization ($M_\perp$) according to the equation:

$$\frac{dM_\perp}{dt} = -\frac{M_\perp}{T_2}$$

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a) If $M_\perp(0)$ is the initial value of the transverse magnetization, immediately after an RF excitation, what is the solution of the equation above?

b) What is the difference between T2 and T2* relaxation?

c) Plot $M_\perp(t)$ vs. t for both the cases of T2 and T2* relaxation. Marking values of $M_{\square}(t)$ at $t = 0$ and $t = \infty$.

d) If you want an image with T2-weighted contrast, would you use a gradient echo (GRE) or spin-echo (SE) sequence? Why?

Solution

a)

$$M_\perp = M_\perp(0)e^{-t/T_2}$$

b) T2 relaxation is the irreversible dephasing among spins due to microscopic interactions with neighboring molecules and nuclei. When there are also macroscopic effects, like magnetic field differences due, for example, to B_0 inhomogeneity and susceptibility variations among tissues, which contribute to the dephasing of the spins, the relaxation is faster and it's called T2*.

c) It's an exponential decay starting at $M_\perp(0)$ for $t = 0$ and faster for the case of T2*. Show illustrative plots.

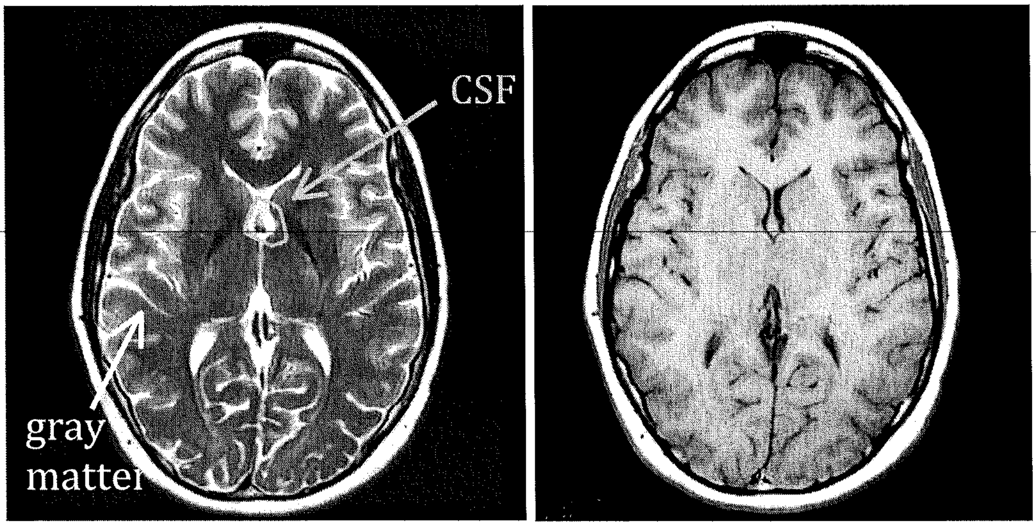
d) SE, as the refocusing pulse "undo" the effect of the macroscopic effects due to magnetic field differences.

Problem 6

a) Complete the table below to show what choice of TE/TR correspond to T1-weighted, T2-weighted, or Proton-Density-weighted contrast (one of the choice is not useful), in the case of a spin-echo pulse sequence. Explain your answers.

	Long TR	Short TR
Short TE		
Long TE		

b) Given that gray matter has T2 ~ 77ms and T1 ~ 760 ms, whereas cerebral spinal fluid (CSF) has T2 ~ 280 ms and T1 ~ 2650 ms, indicate which of the following two images is T1-weighted and which is T2-weighted. Explain your answers.



Solution

a)

	Long TR	Short TR
Short TE	**1) Proton-Density**	**2) T1-weighted**
Long TE	**3) T2-weighted**	**4) Not useful**

1) Short TE → minimize dephasing; long TR → no signal saturation. The combination of the two results in maximum signal from all tissues

2) Short TE → minimize dephasing; short TR → signal of tissues with long enough T1 will be saturated. The combination of the two allows creating a contrast that depends on the T1 of tissues.

3) Long TE → spins of tissues with long enough T2 will diphase (i.e. no signal); long TR → no signal saturation. The combination of the two allows creating a contrast that depends on the T2 of tissues (or T2* for gradient-echo sequences)

4) Long TE → spins of tissues with long enough T2 will diphase (i.e. no signal); short TR → signal of tissues with long enough T1 will be saturated. The combination of the two is not useful because the contrast will be a mix of T1 and T2 effects.

b) The image on the left is T2-weighted, the one on the right is T1-weighted. As CSF has long T1, its signal is saturated by the use of a short TR and therefore it appears dark in T1-weighted image. As CSF has a long T2, it will appear brighter than other tissues with smaller T2 (therefore faster dephasing) on T2-weighted images, which use long TE values.

Problem 7

Match the essential components of an MRI system listed in the left column with one or more descriptions on the right column:

i) Magnet
ii) Gradient and shim coils
iii) RF surface coils

______	A) excite the spins
______	B) polarize the spins
______	C) encode spatial information
______	D) incorporated in the MR system
______	E) detect emitted signal
______	F) compensate for B_0 inhomogeneities

Solution

i) Magnet
ii) Gradient and shim coils
iii) RF surface coils

iii)	A) excite the spins
i)	B) polarize the spins
ii) iii)	C) encode spatial information
i) ii)	D) incorporated in the MR system
iii)	E) detect emitted signal
ii)	F) compensate for B_0 inhomogeneities

Problem 8

a) Explain how slice selection work in MRI.
b) What imaging parameters can be adjusted to control the thickness of the selected slice?

Solution

a) A magnetic field gradient is applied during the RF excitation pulse. The gradient alters the Larmor frequency ω_L of the spins along the direction of the gradient. Only those spins whose Larmor frequency equals the frequency of the RF pulse $\omega_L = \omega_{RF}$ will be excited. Such spins lie in a 'slice' of tissue perpendicular to the gradient

b) The bandwidth of the RF pulse and the strength of the slice selection gradient.

Problem 9

a) Describe the BOLD effect and how it is used in functional MRI (fMRI).
b) What factors limits the spatial resolution of fMRI maps? What source of error is associated with that?

Solution

a) The simplest acceptable answer is that oxygenated blood has different magnetic properties than de-oxygenated blood (optional: the latter is paragnetic therefore distorts locally the magnetic field causing a signal loss), therefore blood oxygenation can be imaged with MRI. This contrast mechanism is exploited in fMRI to map areas of neuronal activation in the brain.

b)

Noise – smaller voxels have lower SNR

Head motion – the smaller the voxels, the more contamination head motion induces

Temporal resolution – the smaller the voxels, the longer it takes to acquire the same volume

(Optional: vasculature)

A typical source of error is partial voluming, i.e. the combination, within a single voxel, of signal contributions from two or more distinct tissue types or functional regions.