EL582/BE620 --- Medical Imaging - I

Physics of Nuclear Medicine

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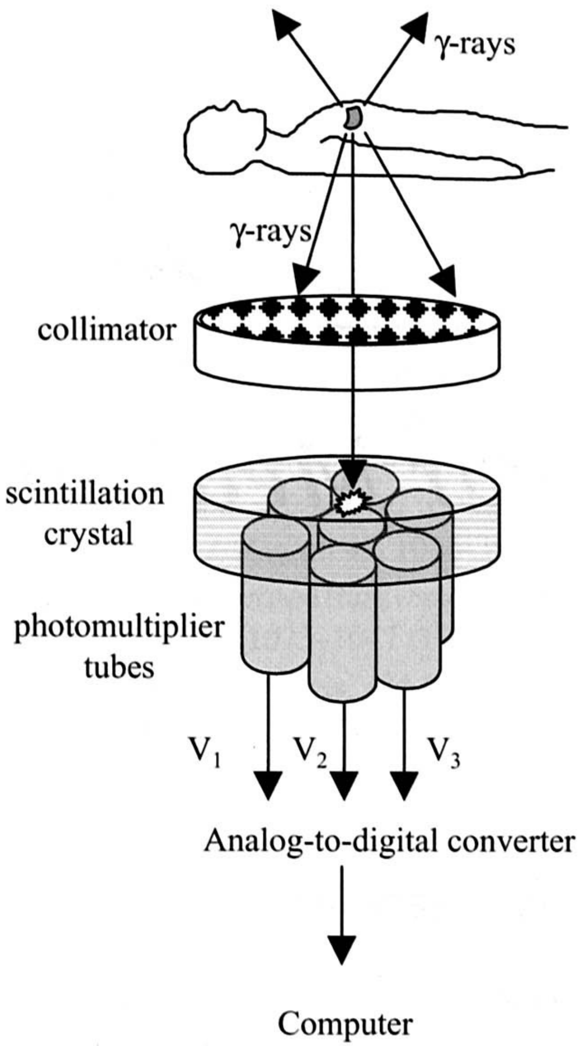
Based on J. L. Prince and J. M. Links, Medical Imaging Signals and Systems, and lecture notes by Prince. Figures are from the textbook.

Lecture Outline

- Atomic structure
- Radioactive Decay
- Decay modes
- Exponential decay law
- Statistical properties of decay
- Radiotracers

What is Nuclear Medicine



- Also known as nuclide imaging
- Introduce radioactive substance into body
- Allow for distribution and uptake/metabolism of compound $\Rightarrow$ *Functional Imaging*!
- Detect regional variations of radioactivity as indication of presence or absence of specific physiologic function
- Detection by "gamma camera" or detector array
- (Image reconstruction)

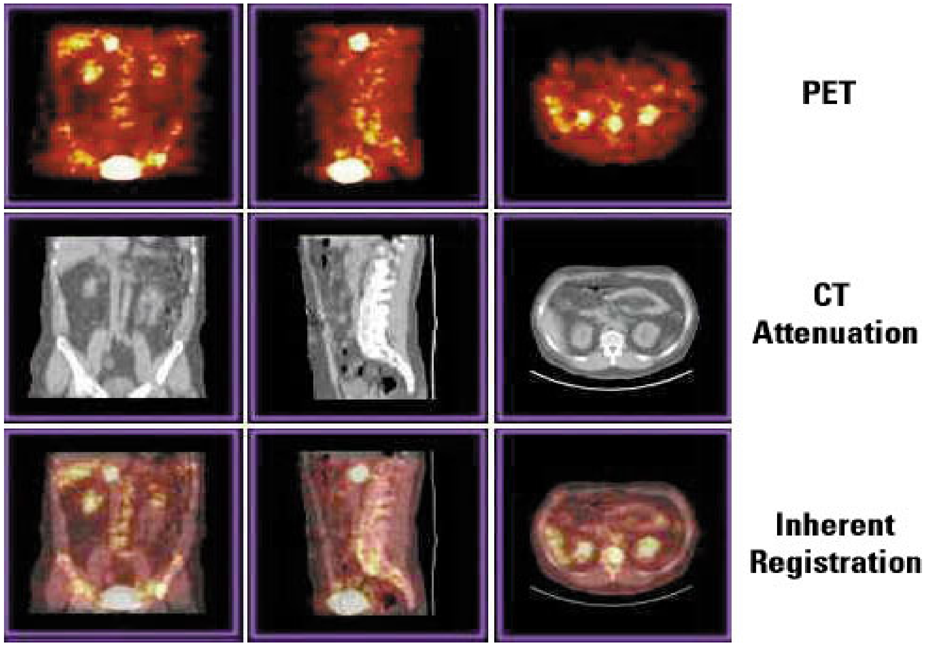
From H. Graber, Lecture Note, F05

What is Nuclear Medicine

- Also known as nuclide imaging
- Steps:
  - Inject radio tracers into the body
  - The radio tracers undergo radioactive decay and generate gammy rays
  - A camera detect gamma rays from the radio tracer after a certain time
- Different physiological functions are imaged by using different radio tracers
- X-ray projection and tomography:
  - X-ray transmitted through a body from an outside source to a detector
- Nuclear medicine:
  - Gamma rays emitted from within a body
  - Emission computed tomography
  - Two popular method:
    - Positron Emission Tomography (PET)
    - Single photon emission computed tomography (SPECT)

Examples: PET vs. CT

- X-ray projection and tomography:
  - X-ray transmitted through a body from a outside source to a detector (transmission imaging)
  - Measuring anatomic structure
- Nuclear medicine:
  - Gamma rays emitted from within a body (emission imaging)
  - Imaging of functional or metabolic contrasts (not anatomic)
    - Brain perfusion, function
    - Myocardial perfusion
    - Tumor detection (metastases)



From H. Graber, Lecture Note, F05

Atomic Structure

- An atom={a nucleus, electrons}
- nucleons = {protons*;* neutrons}
- Nuclide: unique combination of protons and neutrons in a nucleus
- mass number *A* = # nucleons
- atomic number *Z* = # protons = # electrons
- An element is denoted by its A and Z
  - Ex: $^{12}_{6}C$ or C-12

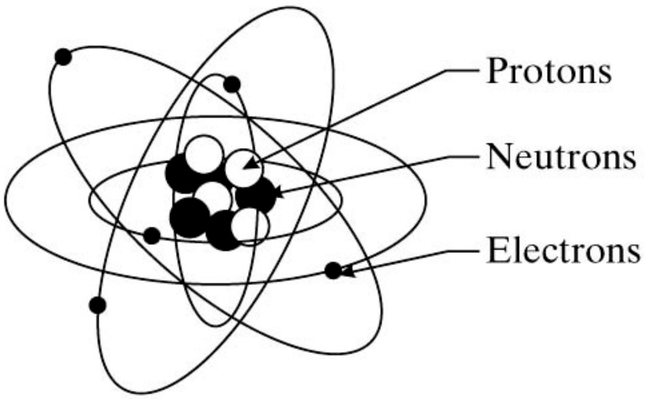


Figure 4.1

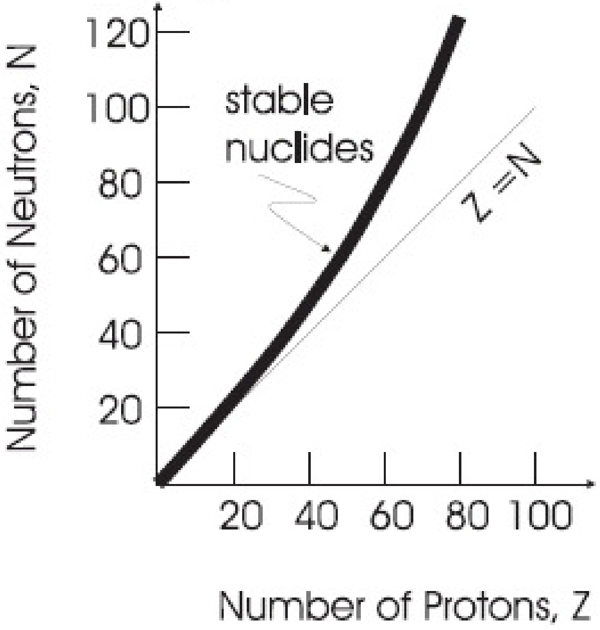
Stable vs. Unstable Nuclides

- Stable nuclides:
  - # neutrons ~= # protons (A ~= 2Z) when Z is small
  - # neutrons > # protons when Z is large
- Unstable nuclides (radionuclides, radioactive atoms)
  - Likely to undergo radioactive decay, which gives off energy and results in a more stable nucleus

Line of Stability

- Nuclides divide into two groups:
  - Non-radioactive — i.e., stable atoms
  - Radioactive — i.e., unstable atoms
- "Line" of stability:

Stability depends on ratio Z:N



Isotopes, etc

- Isotopes: atoms with the same Z but different A
  - E.g. C-12 and C-11
  - Chemically identical
- Isobars: atoms with the same A but different Z
  - Different elements
  - Eg. Carbon-11 and boron-11
- Isotones: atoms with the same number of neutrons but different A
- Isomers: atoms with the same Z and A but with different energy levels (produced after gamma decay)

What is Radioactivity?

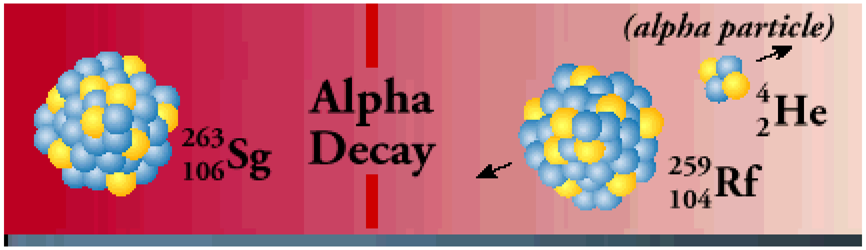
- <u>Radioactive decay:</u> rearrangement of nucleii to <u>lower energy</u> states = greater mass defect
- <u>Parent</u> atom decays to <u>daughter</u> atom
- Daughter has higher binding energy/nucleon than parent
- A radioatom is said to <u>decay</u> when its nucleus is rearranged
- A <u>disintegration</u> is a radioatom undergoing radioactive decay.
- Energy is <u>released</u> with disintegration.

Decay Modes

- Four main modes of decay:
  - alpha particles (2 protons, 2 neutrons)
  - beta particles (electrons)
  - positrons (anti-matter electrons)
  - isomeric transition (gamma rays produced)
- Medical imaging is only concerned with:
  - positrons (PET), and
  - gamma rays (scintigraphy, SPECT)

Alpha Decay

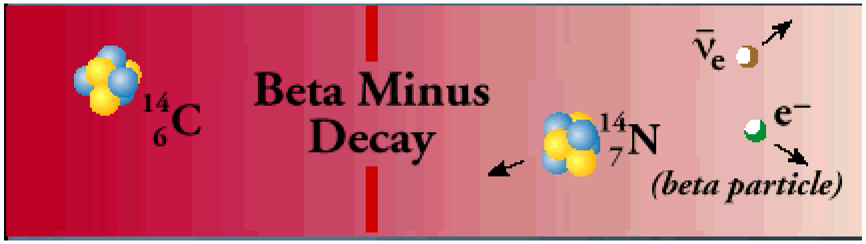
- Alpha decay: the nucleus emits a Helium-4 particle (alpha particle)
  - Alpha decay occurs most often in massive nuclei that have too large a proton to neutron ratio. Alpha radiation reduces the ratio of protons to neutrons in the parent nucleus, bringing it to a more stable configuration.
  - mostly occurring for parent with Z > 82



From: http://www.lbl.gov/abc/wallchart/chapters/03/1.html

Beta Decay

- Beta decay occurs when, in a nucleus with too many protons or too many neutrons, one of the protons or neutrons is transformed into the other.
- Mass number A does not change after decay, proton number Z increases or decreases.
- Beta minus decay (or simply Beta decay): A neutron changes into a proton, an electron (beta particle) and a antineutrino



From: http://www.lbl.gov/abc/wallchart/chapters/03/2.html

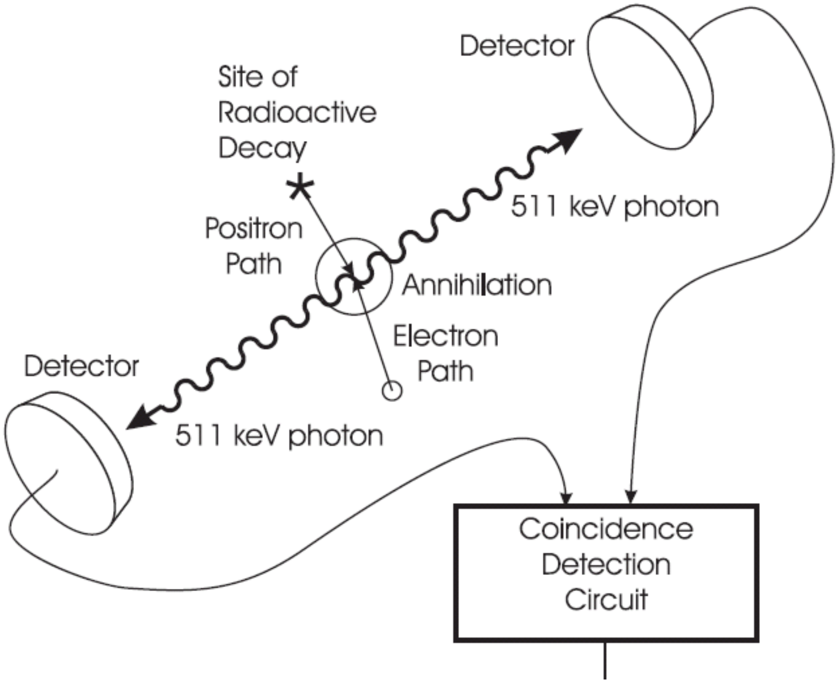
Positron Decay

- Also known as Beta Plus decay
  - A proton changes to a neutron, a positron (positive electron), and a neutrino
  - Mass number A does not change, proton number Z reduces

From: http://www.lbl.gov/abc/wallchart/chapters/03/2.html

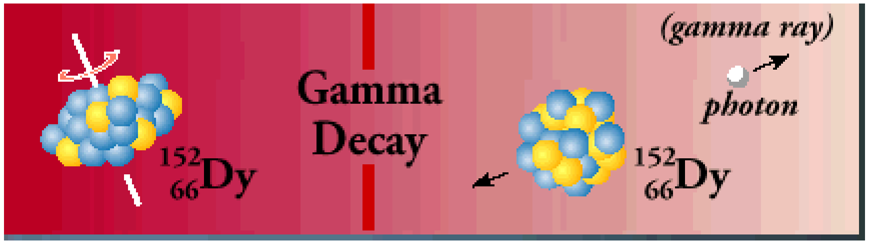
Mutual Annihilation after Positron Decay

- The positron later annihilate a free electron, generate two gamma photons in opposite directions
  - The two photons each have energy 511 KeV, which is the energy equivalent to the rest mass of an electron or positron
  - These gamma rays are used for medical imaging (Positron Emission Tomography), detected using a coincidence detection circuit



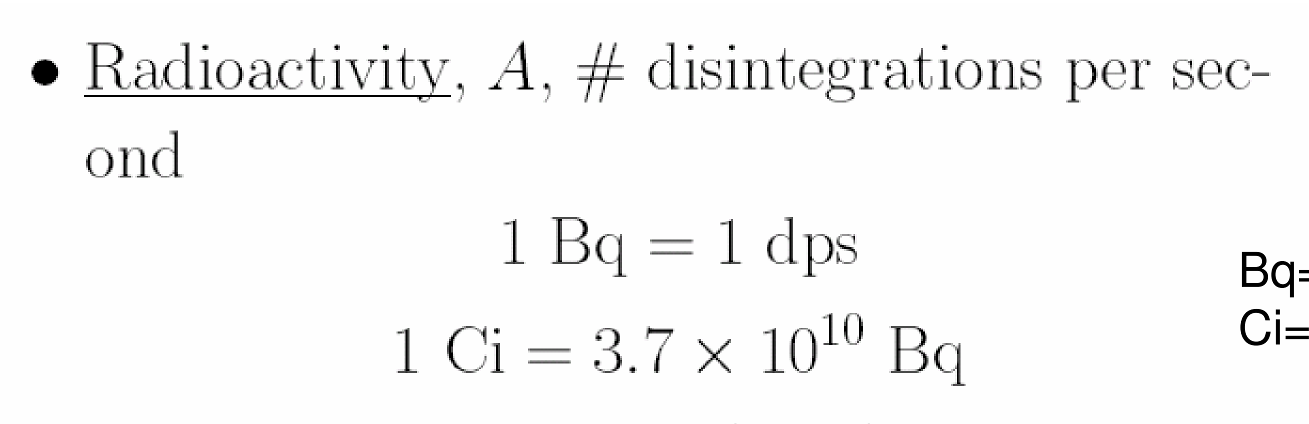
Gamma Decay (Isometric Transition)

- A nucleus (which is unstable) changes from a higher energy state to a lower energy state through the emission of electromagnetic radiation (photons) (called gamma rays). The daughter and parent atoms are isomers.
  - The gamma photon is used in Single photon emission computed tomography (SPECT)
- Gamma rays have the same property as X-rays, but are generated different:
  - X-ray through energetic electron interactions
  - Gamma-ray through isometric transition in nucleus



From: http://www.lbl.gov/abc/wallchart/chapters/03/3.html

Measurement of Radioactivity



- Radioactivity, A, # disintegrations per second

$$1\ \mathrm{Bq} = 1\ \mathrm{dps}$$

$$1\ \mathrm{Ci} = 3.7 \times 10^{10}\ \mathrm{Bq}$$

Bq=Bequerel
Ci=Curie:

(orig.: activity of 1 g of 226Ra)

Naturally occurring radioisotopes discovered 1896 by Becquerel
First *artificial* radioisotopes produced by the Curie 1934 (32P)

The intensity of radiation incident on a detector at range r from a radioactive source is

$$I = \frac{AE}{4\pi r^2}$$

A: radioactivity of the material; E: energy of each photon

Radioactive Decay Law

- N(t): the number of radioactive atoms at a given time
- A(t): is proportional to N(t)

$$A = -\frac{dN}{dt} = \lambda N$$

λ : decay constant

- From above, we can derive

$$\boxed{\begin{aligned} N(t) &= N_0 e^{-\lambda t} \\ A(t) &= A_0 e^{-\lambda t} = \lambda N_0 e^{-\lambda t} \end{aligned}}$$

- The number of photons generated (=number of disintegrations) during time T is
- $$\boxed{\Delta N = \int_0^T A(t)dt = \int_0^T \lambda N_0 e^{-\lambda t} dt = N_0 (1 - e^{-\lambda T})}$$

Half-Life

- Half-life is the time it takes for the radioactivity to decrease by ½.

- Half-life $t_{1/2}$ is defined by

$$\frac{A_{t_{1/2}}}{A_0} = \frac{1}{2} = e^{-\lambda t_{1/2}}$$

- It follows that

$$\boxed{t_{1/2} = \frac{0.693}{\lambda}}$$

Statistics of Decay

- The exponential decay law only gives the expected number of atoms at a certain time t.
- The number of disintegrated atoms over a short time $\Delta t \ll T_{1/2}$ after time t=0 with N_0 atoms follows Poisson distribution

$$\Pr\{\Delta N = k\} = \frac{a^k e^{-a}}{k!}; \quad a = \lambda N_0 \Delta t;$$

λN_0 is called the Poisson rate.

Strictly speaking

$$a = N_0(1 - e^{-\lambda \Delta t})$$

When $\lambda \Delta t$ is small, $e^{-\lambda \Delta t} \approx 1 - \lambda \Delta t, a = N_0 \lambda \Delta t$

Radiotracers: Desired Property

- Decay mode:
  - Clean gamma decay: do not emit alpha or beta articles
  - Positron decay: positron will annihilate with electrons to produce gamma rays
- Energy of photon:
  - Should be high so that photons can leave the body w/ little attenuation
  - Hard to detect if the energy is too high
  - Desired energy range: 70-511 KeV
- Half-life
  - Should not be too short (before detector can capture) or too long (longer patient scan time)
  - Minutes to hours desired
- Half-value-layer (HVL)
  - Thickness of tissue that absorbs half of the radioactivity produced
  - Should be around the dimension of the organ to be imaged
- Monoenergetic
  - Energy sensitive detectors can discriminate the primary photons from scattered ones.

Decay Process Examples

α decay

$$^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}, \quad T_{1/2} \approx 4.5\times10^{9}\,\mathrm{y}$$

β^- decay

$$^{234}_{90}\mathrm{Th} \rightarrow {}^{234}_{91}\mathrm{Pa} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}, \quad T_{1/2} = 24.1\ \mathrm{d}$$

$$^{1}_{0}\mathrm{n} \rightarrow {}^{1}_{1}\mathrm{H} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}, \quad T_{1/2} = 10.6\ \mathrm{m}$$

β^+ decay

$$^{11}_{6}\mathrm{C} \rightarrow {}^{11}_{5}\mathrm{B} + \mathrm{e}^+ + \nu_e, \quad T_{1/2} = 20.38\ \mathrm{m}$$

$$^{10}_{6}\mathrm{C} \rightarrow {}^{10}_{5}\mathrm{B} + \mathrm{e}^+ + \nu_e, \quad T_{1/2} = 19.2\ \mathrm{s}$$

$$^{15}_{8}\mathrm{O} \rightarrow {}^{15}_{7}\mathrm{N} + \mathrm{e}^+ + \nu_e, \quad T_{1/2} = 122\ \mathrm{s}$$

e^- capture

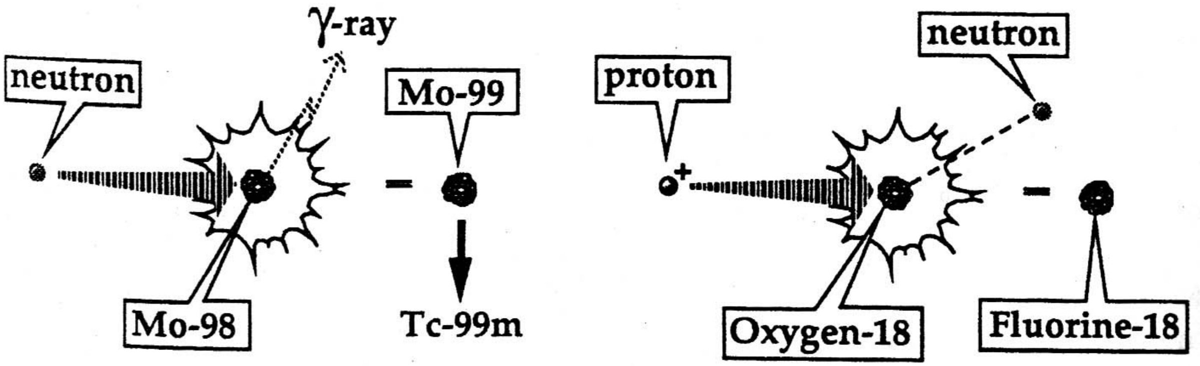
$$^{41}_{20}\mathrm{Ca} + \mathrm{e}^- \rightarrow {}^{41}_{19}\mathrm{K} + \nu_e, \quad T_{1/2} \approx 1\times10^{5}\,\mathrm{y}$$

Most of these naturally occurring processes are not useful for medical imaging applications, with too long Half-time, too short HVL, too high energy.

They can be used as radiotherapeutic agents, if they can be targeted to tumors, to destroy diseased tissue and stops the cancer from proliferating.

Radionuclides in Clinical Use

- Most naturally occurring radioactive isotopes not clinically useful (long $T_{1/2}$, charged particle emission, alpha or beta decay)
- Artificial radioactive isotopes produced by bombarding stable isotopes with high-energy photons or charged particles



- Nuclear reactors (n), charged particle accelerators (Linacs, Cyclotrons)

$$^{99}\mathrm{Mo} \xrightarrow{T_{1/2}=2.5\mathrm{d}} {}^{99m}\mathrm{Tc} + e^- + \bar{\nu}$$

From H. Graber, Lecture Note, F05

The Technetium Generator

- Can be produced from an on-site generator
  - 99^Mo → 99m^Tc → 99^Tc,
- Decay characteristics of 99m^Tc:
  - half life =6.02h, E=140 KeV, HVL=4.6 cm

$$\boxed{{}^{99m}Tc \xrightarrow{T_{1/2}=6\,\mathrm{h}} {}^{99}Tc + \gamma\left(140\,\mathrm{keV}\right)}$$

- Used in more than 90% of nuclear imaging
- More detail: see handout [Webb, sec. 2.5]

Radiopharmaceuticals

- Radionuclide is bound to pharmaceuticals that is specific to metabolic activities (cancer, myocardial perfusion, brain perfusion)
- Gamma emitter
  - ^{99m}Tc-Sestamibi (myocardial perfusion, cancer)
  - ^{99m}Tc-labeled hexamethyl-propyleneamine (brain perfusion)
- Positron emitters
  - ^{11}C, $T_{1/2}$ = 20 min [$^{12}C\ (p,pn)\ ^{11}C$; $^{14}N\ (p,\alpha)\ ^{11}C$]:
    - many organic compounds (binding to nerve receptors, metabolic activity)
  - ^{13}N, $T_{1/2}$ = 10 min [$^{16}O\ (p,\alpha)\ ^{13}N$; $^{13}C\ (p,n)\ ^{13}N$]:
    - NH_3 (blood flow, regional myocardial perf.)
  - ^{15}O, $T_{1/2}$ = 2.1 min [$^{15}N\ (p,n)\ ^{15}O$; $^{14}N\ (d,n)\ ^{15}O$]:
    - CO_2 (cerebral blood flow), O_2 (myoc. O_2 consumption), H_2O (myoc. O_2 consumption & blood perfusion)
  - ^{18}F, $T_{1/2}$ = 110 min [$^{18}O\ (p,n)\ ^{18}F$; $^{20}Ne\ (d,\alpha)\ ^{18}F$]:
    - 2-deoxy-2-[^{18}F]-fluoroglucose (FDG, neurology, cardiology, oncology, metabolic activity)

From H. Graber, Lecture Note, F05

Common Radiotracers

- Gamma Ray Emitters:
  - Iodine-123 (13.3 h, 159 keV)
  - Iodine-131 (8.04 d, 364 keV)
  - Iodine-125 (60 d, 35 keV) (Bad. Why?)
  - Thallium-201 (73 h, 135 keV)
  - Technetium-99m (6 h, 140 keV)
- Positron Emitters:
  - Fluorine-18 (110 min, 202 keV)
  - Oxygen-15 (2 min, 696 keV)

Thyroid function

Kidney function

Most commonly used

Oxygen metabolism

Summary

- Nuclear medicine relies on radiation (gamma rays) generated through radioactive decay
- Radioactive decay is the process when a unstable nuclide is changed to a more stable one
  - Four modes of decay, generating alpha particles, beta particles, positrons and gamma rays respectively
- Radioactivity follows an exponential decay law, characterized by the decay constant or the half-life
- Desired properties for radio tracers
- Common radiotracers in nuclear medicine

Reference

- Prince and Links, Medical Imaging Signals and Systems, Chap 7.
- "Guide to the Nuclear Wallchart", Chap 3.

http://www.lbl.gov/abc/wallchart/outline.html

- Recommended readings:
  - K. Miles, P. Dawson, and M. Blomley (Eds.), *Functional Computed Tomography* (Isis Medical Media, Oxford, 1997).
  - R. J. English, *SPECT: Single Photon Emission Computed Tomography: A Primer* (Society of Nuclear Medicine, Reston, VA, 1995).
  - M. Reivich and A. Alavi (Eds.), *Positron Emission Tomography* (A. R. Liss, NY, 1985).

Homework

- Reading:
  - Prince and Links, Medical Imaging Signals and Systems, Ch. 7.
  - Handouts from [Webb]
- Note down all the corrections for Ch. 7 on your copy of the textbook based on the provided errata.
- Problems for Chap 7 of the text book
  - P7.1
  - P7.2
  - P7.4
  - P7.6
  - P7.7 (assume the energy of the photons is E)
  - P7.9