Medical Imaging I, Spring 2012 Midterm Solution.

1. (a) X-ray source is located outside of the ~~body~~ body.
   The photon intensity is measured.
   The linear attenuation coefficient is revealed.

   (b) The source is ~~at~~ inside the body.
   The photon intensity is measured.
   Radioactive distribution in the tissue is revealed.

2. (a) Maximum spacing $\tau \leq \frac{1}{2 f_{max}}$ based on the nyquist frequency theory.

   (b) $M = \frac{D}{\tau}$, $N = \frac{D}{\tau} \geq 2D f_{max}$

   (c) $K = \frac{2D}{\tau}$ (projection need to be zero padded to avoid aliasing)

3. (a) For SPECT, radio tracer undergo gamma decay.
   Yes. Collimator is needed for SPECT to reduce the scattering artifact.

   (b) For PET, radio tracer undergo positron decay.
   No. Collimator is not needed, since PET use annihilation coincidence detection (ACD) to reject scattered gamma rays.

   (c) The radio tracers that have short half-life (radio tracers used in PET) need to be produced at the ~~a~~ imaging facility; while the radio tracers that have long half-life (radio tracers used in SPECT) can be manufactured at offsite locations and shipped to imaging facility.

4. ① CT reconstruction:

Given the photon intensity $I(\theta_m, l_n)$, $g(\theta_m, l_n) = -\ln\left(\frac{I(\theta_m, l_n)}{I_0}\right) = \int_{-\infty}^{\infty} \mu(x(s), y(s))\,ds$

CT image is reconstructed by convolution backprojection:

$$\mu(x,y) = \phi \sum_{\theta_m} \left[C(l_n) * g(\theta_m, l_n) \right]_{l_n = x\cos\theta_m + y\sin\theta_m}$$

② PET reconstruction:

The data $I(\theta_m, l_n)$ acquired in CT imaging can be used to correct the PET data:

$$\Phi_c(\theta_m, l_n) = \frac{\Phi(\theta_m, l_n)}{K \exp\{-\int_{-R}^{R} \mu(x(s), y(s))\,ds\}} \cancel{= \int_{-R}^{R} A(x(s), y(s))\,ds}$$

$$= \frac{\Phi(\theta_m, l_n)}{K \cdot I_{CT}(\theta_m, l_n)} = \int_{-R}^{R} A(x(s), y(s))\,ds$$

And then by convolution backprojection:

$$A(x,y) = \sum_{\theta_m} \left[c(l_n) * \Phi_c(\theta_m, l_n) \right]_{l_n = x\cos\theta_m + y\sin\theta_m}$$

5. Sensitivity: $\frac{\int_{t}^{\infty} P_D(x)}{\int_{-\infty}^{\infty} P_D(x)} = \int_{t}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma_d} \exp\{-(x-\mu_d)^2/\sigma_d^2\}$

Specificity: $\frac{\int_{-\infty}^{t} P_N(x)}{\int_{-\infty}^{\infty} P_N(x)} = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}\,\sigma_n} \exp\{-(x-\mu_n)^2/\sigma_n^2\}$

When increasing t, sensitivity is decreased while specificity is increased.

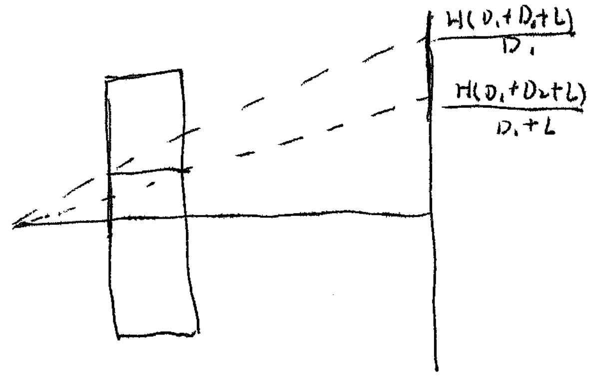
One possible way to find a tradeoff is to maximize the diagnostic accuracy, which equals $\frac{\text{true positive} + \text{true negative}}{\text{all cases}}$.

6.

① When $y \le 0$ or $y > \dfrac{H(D_1+D_2+L)}{D_1}$

$$I(y) = \frac{I_0}{4\pi (D_1+D_2+L)^2} \cos^3\theta \, e^{-\mu_1 L/\cos\theta}$$

where $\cos\theta = \dfrac{D_1+D_2+L}{\sqrt{y^2+(D_1+D_2+L)^2}}$



② When $\dfrac{H(D_1+D_2+L)}{D_1+L} < y \le \dfrac{H(D_1+D_2+L)}{D_1}$

$$I(y) = \frac{I_0}{4\pi (D_1+D_2+L)^2} \cos^3\theta \, e^{-\mu_1\left(D_1+L-\frac{H(D_1+D_2+L)}{y}\right)/\cos\theta - \mu_2\left(\frac{H(D_1+D_2+L)}{y}-D_1\right)/\cos\theta}$$

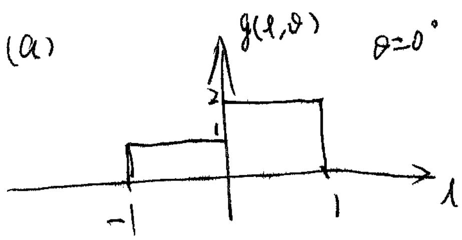
where $\cos\theta = \dfrac{D_1+D_2+L}{\sqrt{y^2+(D_1+D_2+L)^2}}$

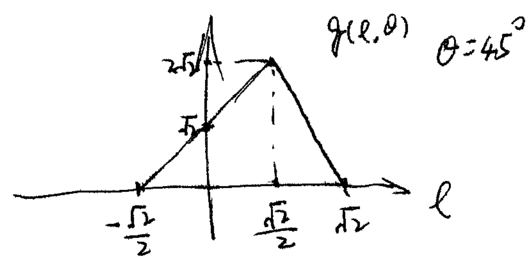
③ When $0 < y \le \dfrac{H(D_1+D_2+L)}{D_1+L}$

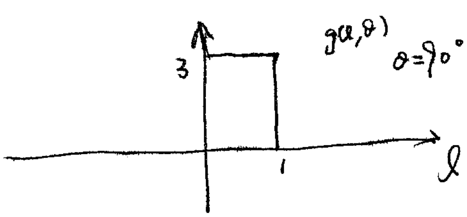
$$I(y) = \frac{I_0}{4\pi (D_1+D_2+L)^2} \cos^3\theta \, e^{-\mu_2 L/\cos\theta}$$

where $\cos\theta = \dfrac{D_1+D_2+L}{\sqrt{y^2+(D_1+D_2+L)^2}}$

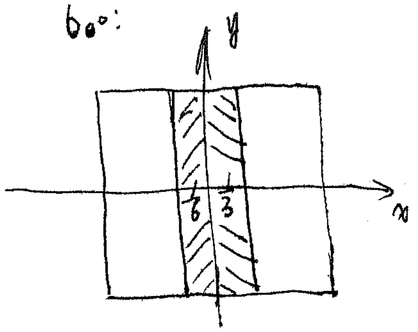
7. (a)



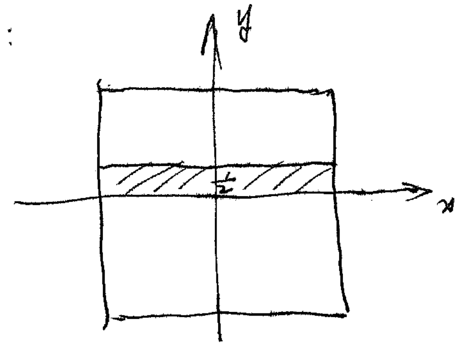




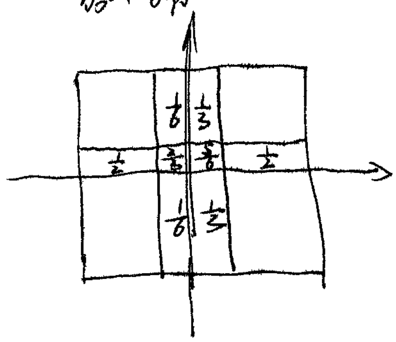
(b) b_{0°:



b_{90°:



$b_{0^\circ} + b_{90^\circ}$



(c)

$$F(\rho\cos\theta, \rho\sin\theta) = G(\rho,\theta)$$

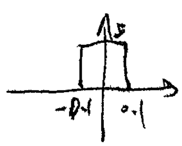
$$g(l,0) = \mathrm{rect}(l+\tfrac{1}{2}) + 2\,\mathrm{rect}(l-\tfrac{1}{2})$$

$$F(\rho\cos 0^\circ, \rho\sin 0^\circ) = G(\rho, 0^\circ) = \mathrm{sinc}(\rho)\, e^{j\pi\rho} + 2\,\mathrm{sinc}(\rho)\, e^{-j\pi\rho}$$

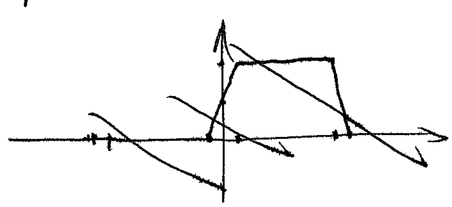
(d)

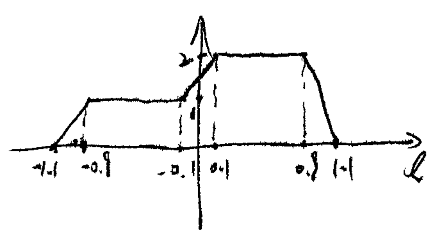
$$\hat{g}(l,\theta) = g(l,\theta) * h(l)$$

$h(l) =$



$\hat{g}(l,0) =$





(e) We know that $\hat{f}(x,y) = f(x,y) * R^{-1}\{h(\ell)\}$, where $h(\ell) = \mathrm{rect}(\frac{\ell}{0.2})$

Since $h(\ell)$ is circularly symmetric, $\hat{f}(x,y) = f(x,y) * h(r)$,

where $h(r) = \mathcal{H}^{-1}\{H(\rho)\}$, $H(\rho) = 0.2\ \mathrm{sinc}(0.2\rho)$

$$= \frac{2\,\mathrm{rect}(5r)}{\pi\sqrt{0.2^2 - 4r^2}}$$

8. (a) $A_t = A_1 e^{-\lambda t}$, where $\lambda = \frac{0.693}{T_1}$

$$\Phi_A = \int_{-1}^{0} \frac{A_t}{4\pi(3-s)^2} \exp\{-\mu_1(-s) - 2\mu_2 - \mu_3\}\, ds + \int_0^2 \frac{A_t}{4\pi(3-s)^2} \exp\{-\mu_2(2-s) - \mu_3\}\, ds$$

$$\Phi_B = \int_{-1}^{0} \frac{A_t}{4\pi(3+s)^2} \exp\{-\mu_1(1+s) - 2\mu_3\}\, ds + \int_0^2 \frac{A_t}{4\pi(3+s)^2} \exp\{-\mu_2 s - \mu_1 - 2\mu_3\}\, ds$$

(b) $A_t = A_2 e^{-\lambda t}$ where $\lambda = \frac{0.693}{T_2}$

$$\Phi_{AB} = \int_{-1}^{2} A_t\, ds \cdot \exp\{-\mu_1 - 2\mu_2 - 3\mu_3\} = 3A_t \cdot \exp\{-\mu_1 - 2\mu_2 - 3\mu_3\}$$