EL-GY 6813 / BE-GY 6203 / G16.4426
Medical Imaging

Physics of Radiography

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Based on Prince and Links, Medical Imaging Signals and Systems and Lecture Notes by Prince. Figures are from the book.

Lecture Outline

- Atomic structure and ionization
- Particulate Radiation
  - Focusing on energetic electron interaction
- EM Radiation
  - Photoelectric
  - Compton scattering

    -> Likelihood of each phenomenon
  - EM radiation measurement
  - Attenuation of radiation
- Radiation Dosimetry
  - Exposure, dose

Atomic Structure

- An atom={a nucleus, electrons}
- Nucleus composed of nucleons = {protons; neutrons}
- mass number *A* = # nucleons
- atomic number *Z* = # protons = # electrons
  - Define an element with a particular symbol: H, C, etc.
  - An element is denoted by its A and Z
  - Ex: $^{12}_{6}C$ or $C\text{-}12$

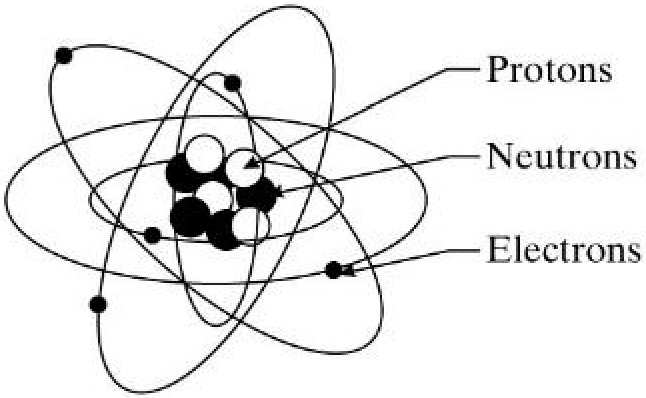


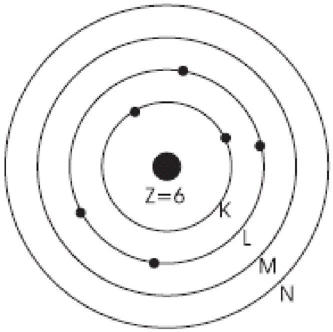
Figure 4.1

Stable vs. Unstable States

- Stable nuclides:
  - # neutrons ~= # protons (A ~= 2Z)
- Unstable nuclides (radionuclides, radioactive atoms)
  - Likely to undergo radioactive decay, which gives off energy and results in a more stable nucleus

Orbits of Electrons

Shell Number n	Shell Label	# Electrons $2n^2$
1	K	≤ 2
2	L	≤ 8
3	M	≤ 18
4	N	≤ 32



Ground state: electrons are in the lowest orbital shells and within the lowest energy quantum states within each shell

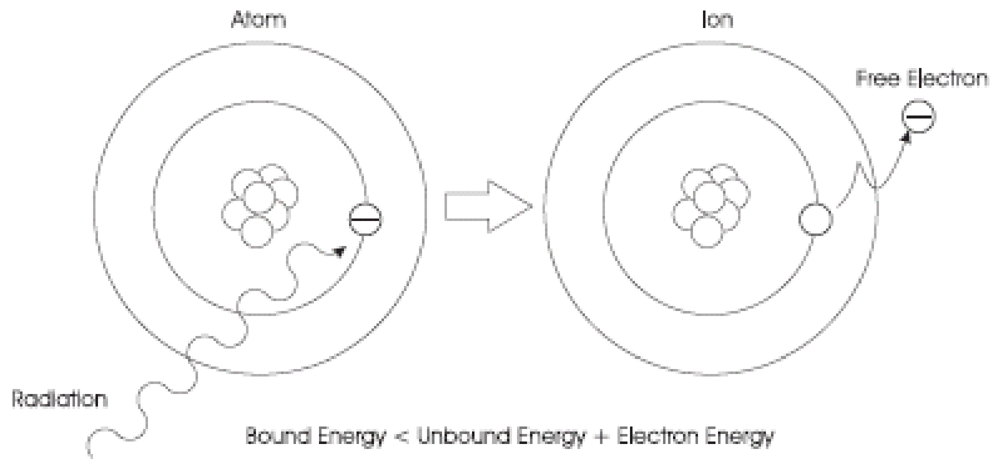
Electron Binding Energy

- A free electron has higher energy than when it is bounded to an nucleus in an atom
- Binding energy = energy required to free electrons from their atomic orbits
  - Depends on the element to which the electron is bound and the shell within which it resides in ground state
  - Sufficient to consider “average” binding energy of a given atom
- One electron volt (eV) = kinetic energy gained by an electron when accelerated across one volt potential
  - 1 eV = 1.6 x 10^{-19} Joule
- Binding energies of typical elements:
  - hydrogen = 13.6 eV
  - Air: 29 eV
  - Lead: 1 KeV
  - Tungsten: 4 KeV (considered a “heavy” element)

Electronvolt (**eV**) is a unit of energy equal ~1.602×10^{-19} J. It is the amount of energy gained (or lost) by the charge of a single electron moving across a 1-V electric potential difference -> 1 volt (= 1 J/C) multiplied by the elementary charge (*e*, or ~1.602×10^{-19} C). 1 eV = 1.602×10^{-19} J.

Ionization and Excitation

- Ionization is "knocking" an electron out of an atom
  - Creates a free electron + ion (an atom with +1 charge)
  - Occurs when radiated with energy above the electron binding energy
- Excitation is "knocking" an electron to a higher orbit
  - When the radiation energy is lower than the binding energy
- After either ionization or excitation, an atom has higher energy

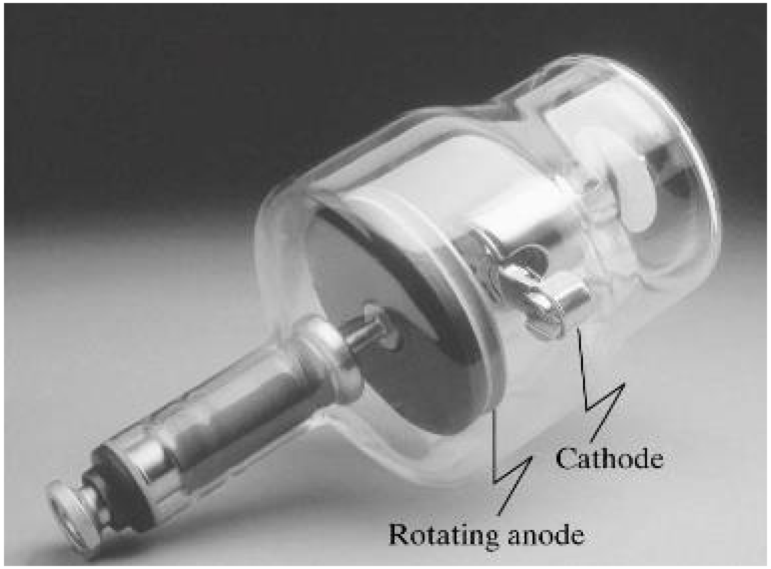


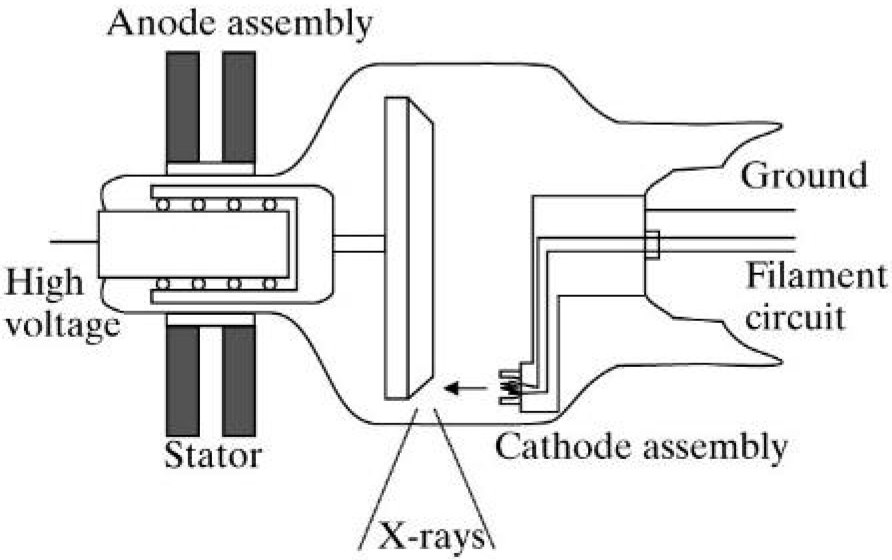
Characteristic Radiation

- What happens to ionized or excited atom?
  - Return to ground state by rearrangement of electrons
  - Causes atom to give off energy
  - Energy given off as characteristic radiation
    - infrared
    - light
    - x-ray

Example

- Consider an electron accelerated through an X-ray tube where the anode is made of tungsten. If the anode is held at 120 KV, what is the maximum number of tungsten atoms that can be ionized?
- Solution:
  - The electron will have 120 KeV kinetic energy when reaching the anode, by definition of eV
  - The average binding energy of tungsten = 4 KeV
  - # ionized atoms = 120/4=30





Ionizing Radiation

- Radiation with energy > 13.6 eV is <u>ionizing</u>
- Energy required to ionize:
  - air ≈ 34 eV
  - lead ≈ 1 keV
  - tungsten ≈ 4 keV

  These are average binding energies.
- Radiation energies in medical imaging

  30 keV–511 keV

  can ionize 10–40,000 atoms

Two Types of Ionizing Radiation

- Particulate
- Electro-magnetic (EM)

Particulate Radiation

- Radiation by any particle (proton, neutron or electron) if it possesses enough kinetic energy to ionize an atom

$$\text{Kinetic Energy} = \text{the energy gained due to motion}$$

$$\text{Mass of a moving particle}: \ m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

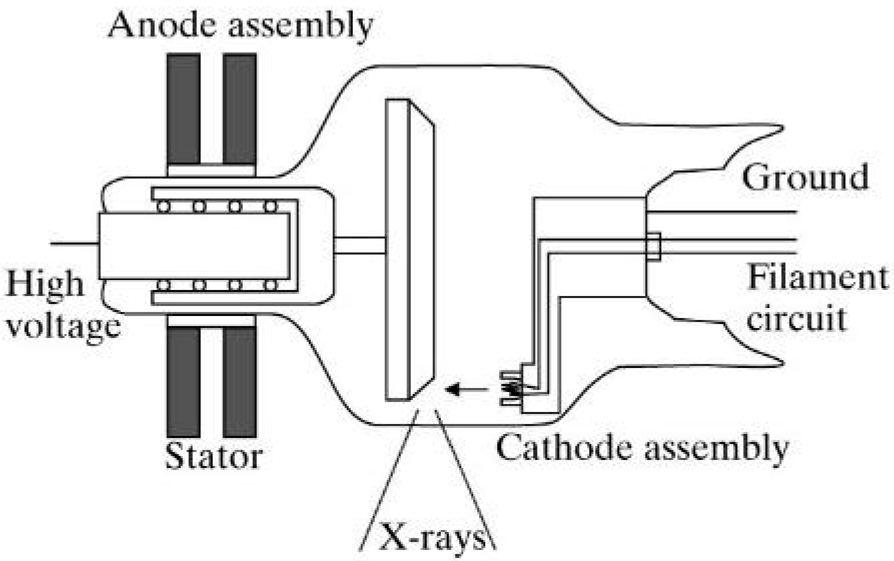
$$\text{Energy vs. mass}: E = mc^2$$

$$\text{Kinetic Energy}: \quad KE = E - E_0 = (m - m_0)c^2$$

$$\text{When } v << c, \frac{1}{\sqrt{1 - v^2/c^2}} \approx 1 + \frac{1}{2} v^2/c^2, KE = \frac{1}{2} m_0 v^2$$

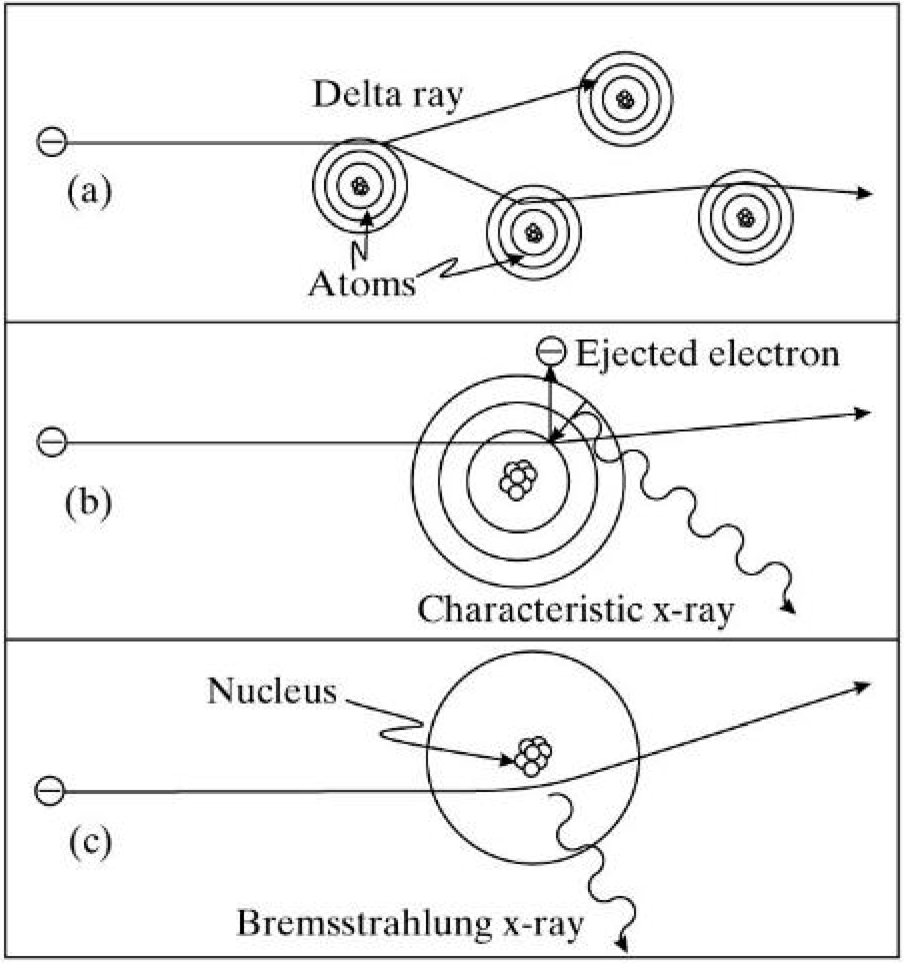
Particulate Radiation by Energetic Electrons

- We are only concerned with an electron accelerated in a X-ray tube here
  - An electron accelerated across a tube with 100 KV potential possesses 100 KeV kinetic energy



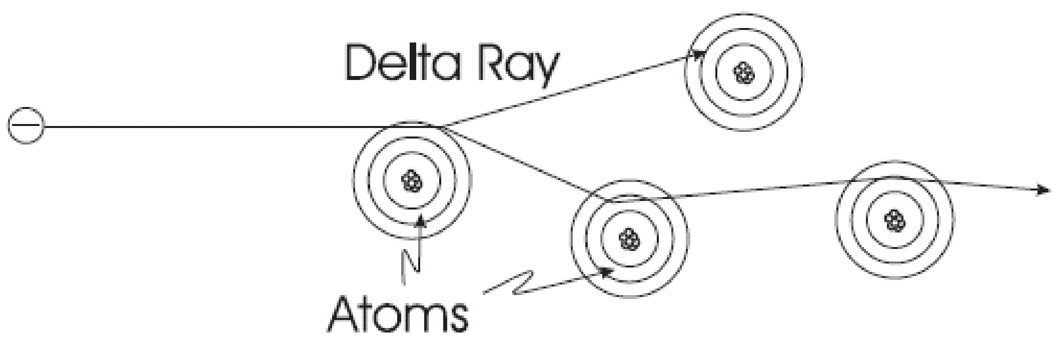
Energetic Electron Interactions

- Two primary interactions
  - Collisional transfer
    - Most common
    - Produces heat
  - Radiative transfer
    - Produces x-ray
    - Characteristic radiation
      - Collide with K-shell
    - Bremsstrahlung radiation
    ("Braking or deceleration radiation")
      - Collide with nucleus
      - More common than characteristic radiation



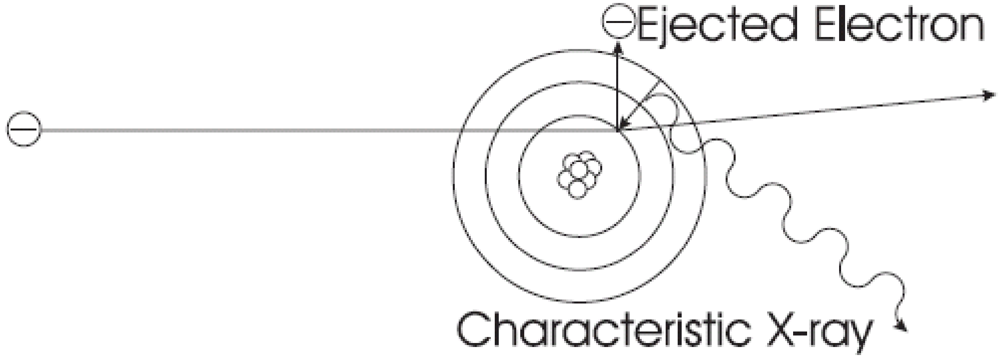
Collisional Transfer

- The energetic electron collides with an atom in the target
- Typically, a small fraction of the kinetic energy of the electron is transferred to another electron in the atom
  - As the affected atom returns to its original state, infrared radiation (heat) is generated
- Occasionally, a large fraction of the incident energy is transferred to another electron, the newly freed electron may form a delta ray
- The incident electron's path may be redirected, and many other subsequent interactions may occur, until the kinetic energy of the incident electron is exhausted



Characteristic X-Ray

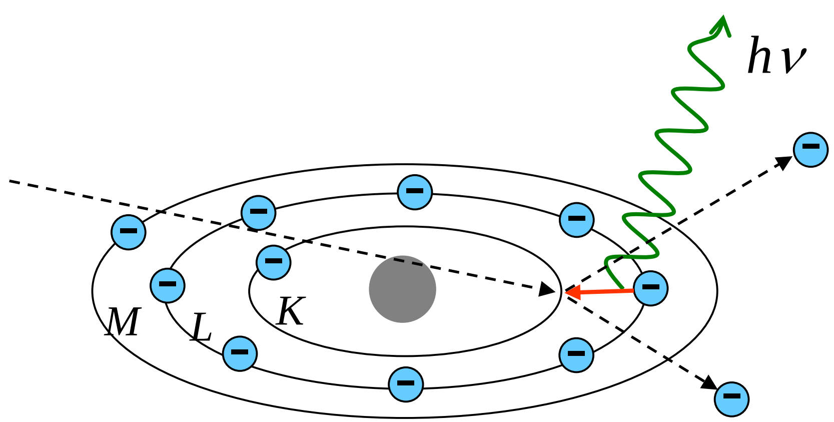
- The incident electron collides with a K-shell electron, exciting or ionizing the atom, leaving a hole in that shell.
  - As the atom returns to its ground state, the k-shell hole is filled by a higher shell electron
  - The loss of energy creates an EM photon, known as Characteristic x-ray
  - The energy of the x-ray photon = difference between the energy of the two shells (element dependent)

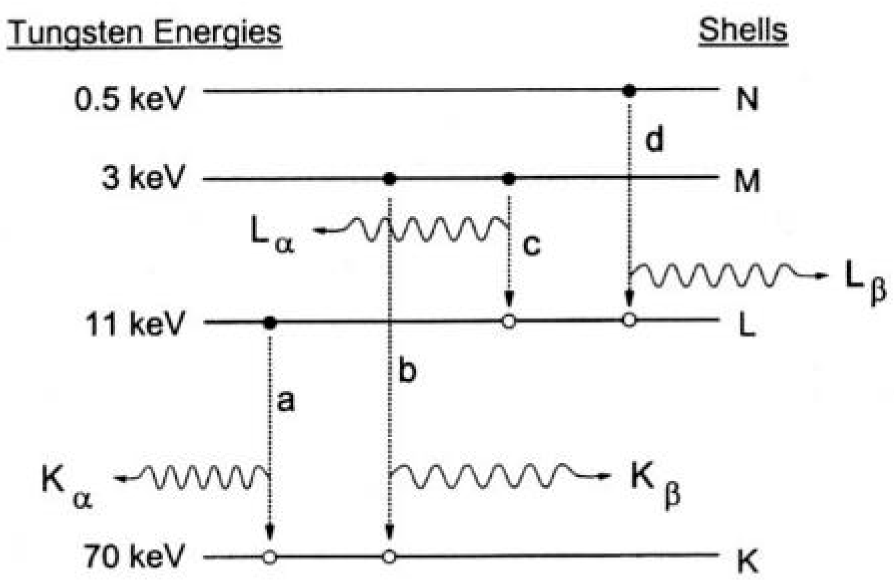


Characteristic radiation

- Caused by removal of inner shell electrons and subsequent filling of hole with electrons from higher shell. The shell-energy difference determines the energy of characteristic rays
- Lines are named after the lower shell involved in the process; the upper shell involved is denoted by Greek letters:

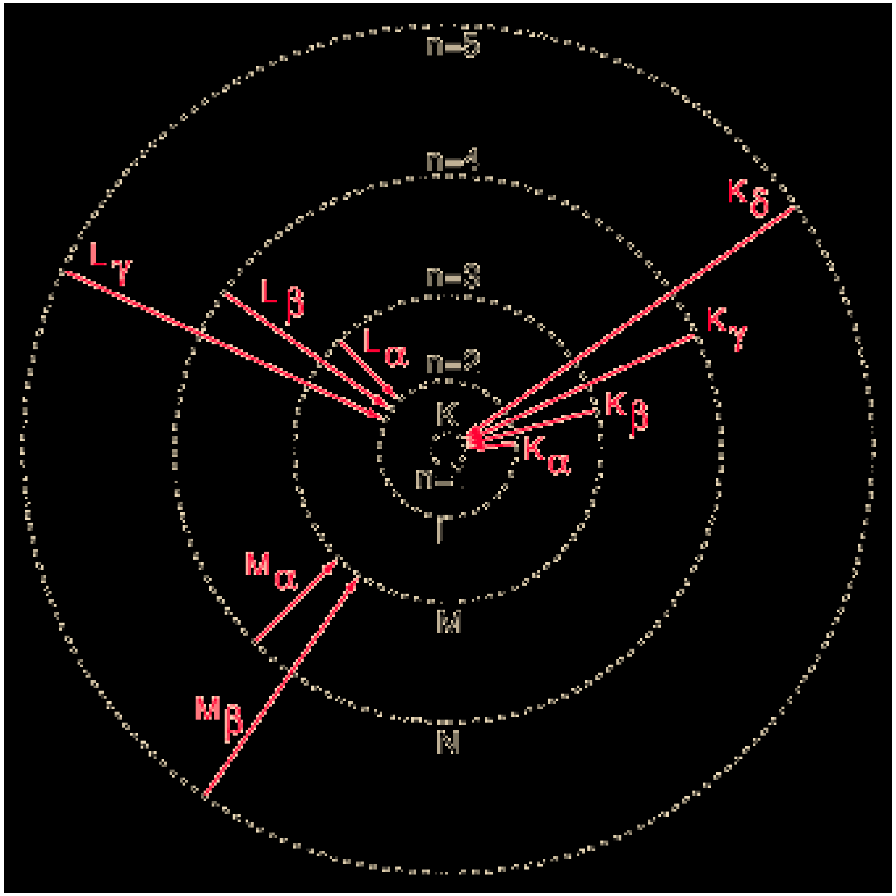
$$\Delta n = 1: \alpha \text{ transitions; } \Delta n = 2: \beta \text{ transitions;}$$





[From Graber, Lecture Note for BMI1-FS05]

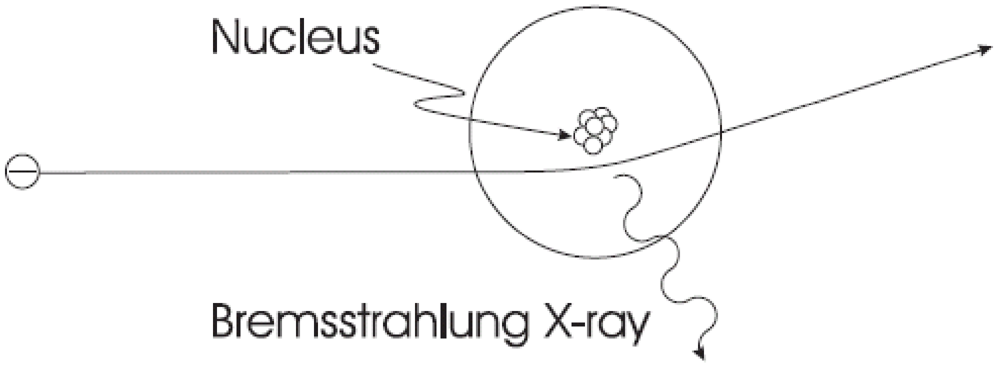
Different types of characteristics rays



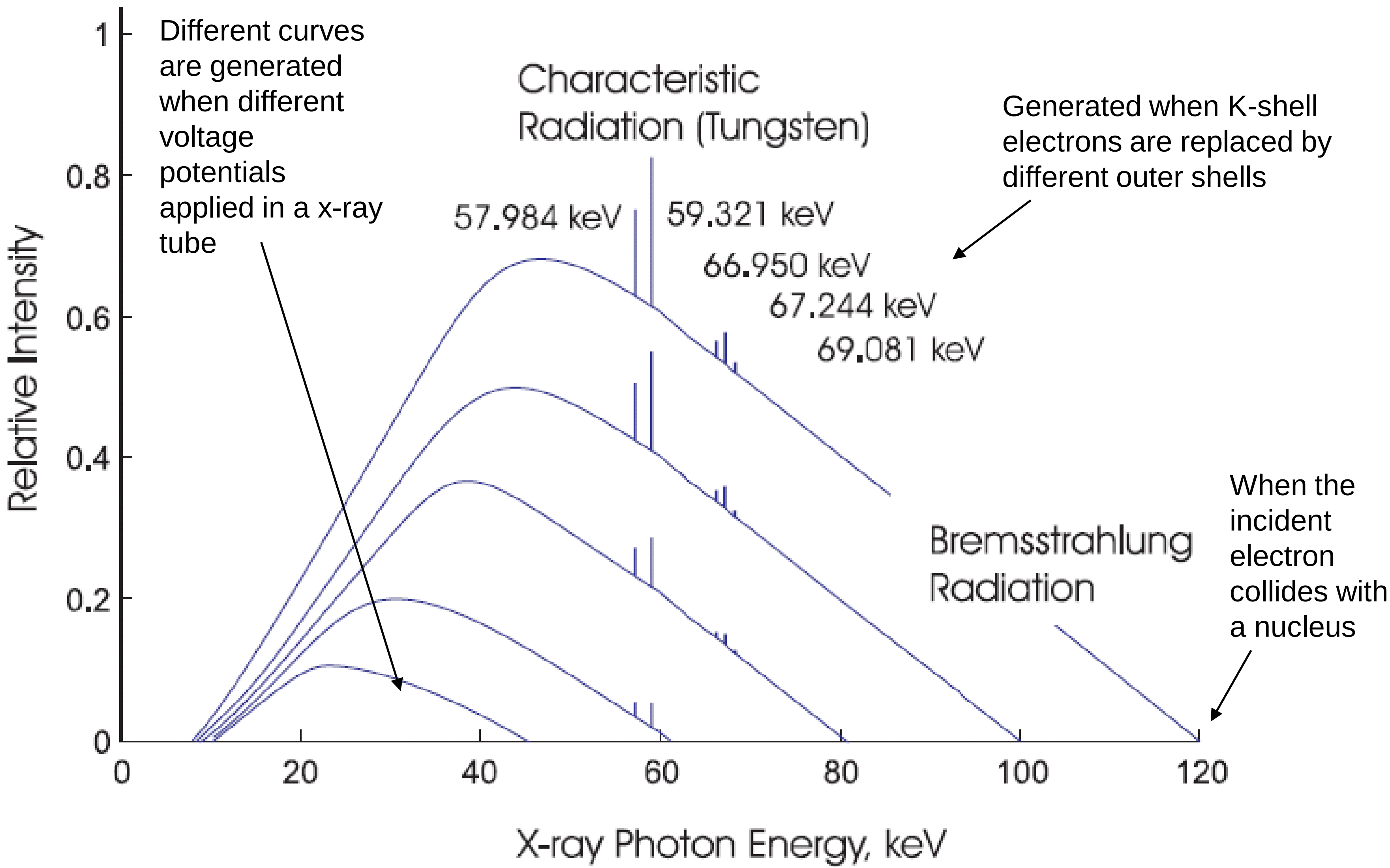
From http://hyperphysics.phy-astr.gsu.edu/Hbase/quantum/xterm.html#c1

Bremsstrahlung Ray

- As the incident electron approaches the nucleus of an atom, the positive charge of the nucleus causes the incident electron to bend around the nucleus and decelerates
  - The loss of energy leads to the Bremsstrahlung x-ray (energy vary over a continuous range, depending on the speed loss)
- Occasionally when the incident electron collides with the nucleus, the electron is annihilated, emitting a photon with an energy equal to the kinetic energy of the incident electron (highest possible energy)
- Primary source of x-rays from an x-ray tube



Spectrum of X-Ray



EM Radiation

- EM radiation comprises an electric wave and a magnetic wave traveling at right angles to each other
- Typical EM waves:
  - Non ionizing: radio, microwaves, infrared, visible light, ultraviolet
  - Ionizing: X-rays, gamma rays
- Energy of a photon of an EM wave with frequency *v:*

$$E = h\nu$$

$$\text{Planck's constant } h = 4.14 \times 10^{-15} \text{ eV-sec}$$

EM Waves for Medical Imaging

- X-rays and Gamma rays:
  - Have energy in the KeVs to MeVs -> Ionizing Radiation
  - used in X-ray/CT and nuclear medicine respectively
  - X-rays are created by interaction of energetic electrons with atoms
  - Gamma rays are created in the nuclei of atoms due to radioactive decay or characteristic radiation, typically have higher energy than X-rays
- Radio waves
  - Used to stimulate nuclei in MRI to generate EM radiation
- Visible light
  - Used in radiography to improve the efficiency of photographic film to detect X-rays
- See Table 4.2 for more details

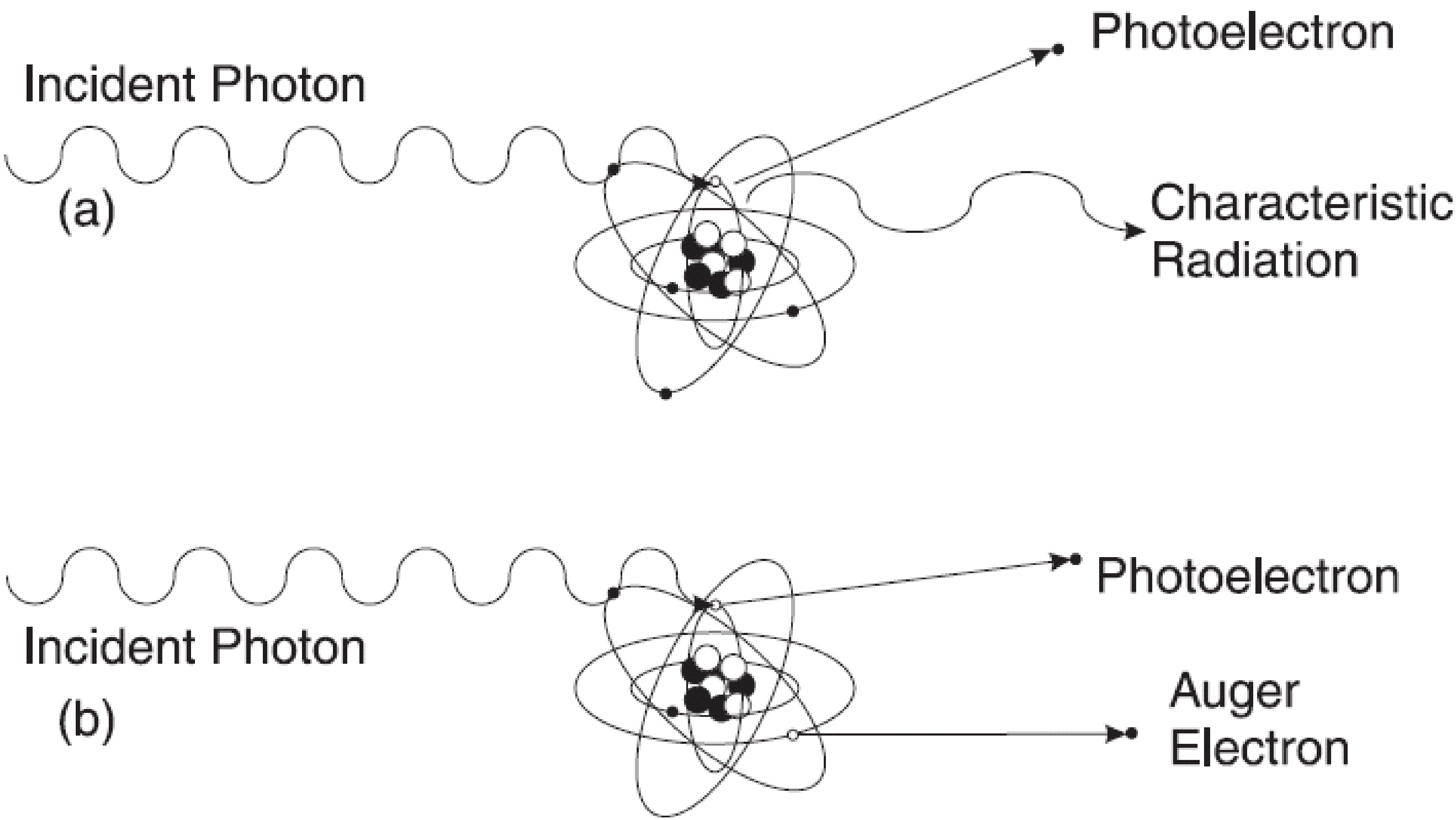
EM Radiation Interactions

- Two main interactions
  - Photoelectric effect
    - The incoming photon is completely absorbed and ejecting K-shell or L-shell electrons, producing characteristic x-ray
  - Compton scattering
    - The incoming photon changes its direction

Photoelectric Effect

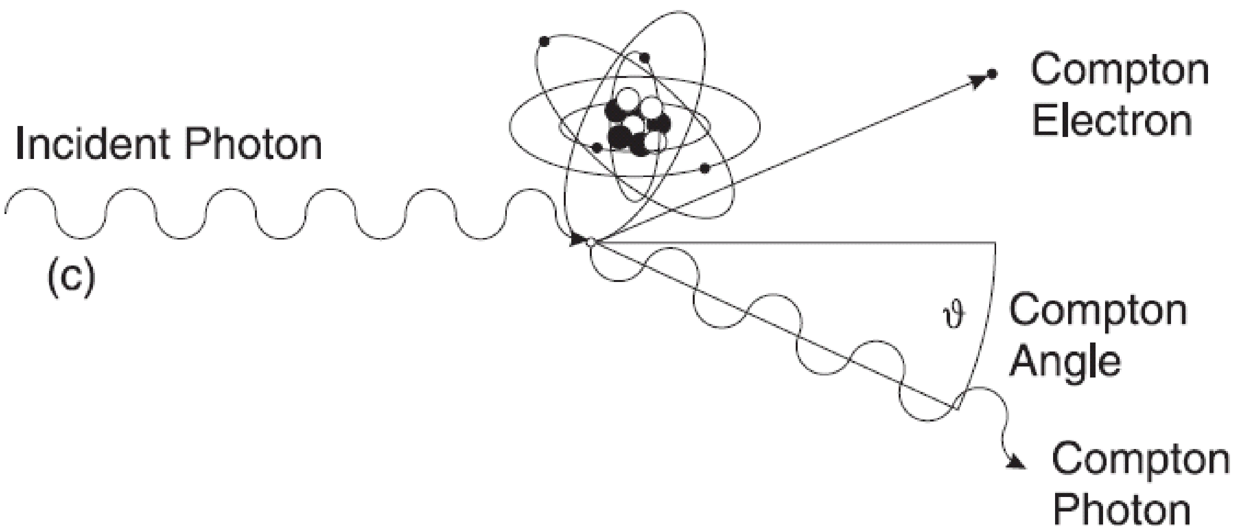
- An incoming photon interacts with the nucleus of an atom, causing ejection of a K-shell or L-shell electron (photoelectron)
  - Atom completely absorbs incident photon and all energy is transferred
  - The photoelectron propagates away with energy $E_{e^-} = h\nu - E_B$
  - The affected atom produces characteristic x-ray, while outer electrons fill the K-shell.
  - Sometimes the characteristic x-ray transfers its energy to an outer electron (called Auger electron)
- Both photo electron and Auger electron are energetic electrons that can interact with the matter as discussed before

Photoelectric Effect



Compton Scattering

- An incoming photon ejects an outer shell electron, yielding a Compton electron
- The incident photon loses its energy and changes its direction (Not completely absorbed by the atom!)
- The scattered photon is called Compton photon



- The energy of the scattered photon depends on the scatter angle

$$E' = \frac{E}{1 + E(1 - \cos\theta)/(m_0 c^2)}$$

- m_0 is rest mass of electron
- $m_0 c^2 = 511$ keV

  – The maximum energy loss occurs when the photon is deflected backward ($\theta = 180^o$), backscattering
  – When E is higher, more photons scatter forward
  – The kinetic energy of the Compton electron = E-E'

Which interaction is better?

- Photoelectric effect helps to differentiate different human tissues/organs
- Compton scattering causes incident photons to deviate from straight path, and causes unnecessary exposure of x-ray to untargeted areas
- In medical imaging, we want to increase the likelihood of photoelectric events, while minimizing Compton scattering

Probability of Photoelectric Effect

- Recall that a photoelectric event happens when incident photons interact with the Coulomb field of the nucleus of an atom
- More likely when colliding with an atom with more protons (i.e., larger Z number)
- Less likely when incident photons have higher energy (higher frequency)

$$\text{Prob}[\text{photoelectric event}] \propto \frac{Z_{\text{eff}}^4}{(h\nu)^3}$$

  - The probability increases abruptly when the photon energy rises above the binding energy of L-shell or K-shell electrons (so as to eject the electrons), then begins to diminish
  - Rationale behind the use of "contrast agent"
  - Z numbers: soft tissue: 7.4 soft, bone: 13.8
  - More contrast with low energy x-ray
  - Photoelectric effect dominate over Compton scattering at low energy

Probability of Compton Scattering

- Recall that Compton scattering occurs when an incident photon collides with outer shell electrons
- Likelihood proportional to the number of electrons per kilogram of the material (the electron density or ED)
- Relatively independent of incident photon energy in biological materials

$$\text{Prob}[\text{Compton event}] \propto \text{ED}$$

$$ED = \frac{N_A Z}{W_m}$$

N_A : Avogadro's number (atoms/mole)

Z : atomic number (electrons/atom)

W_m : molecular weight (grams/mole)

- ED is approximately constant for various biological material, ~ $3*10^{26}$, except for hydrogen ($6*10^{26}$)
- **Does not provide tissue contrast**

Relative Likelihood

- Compton scattering is equally likely in various materials and invariant of incident energy
- Photoelectric effect is more likely in high Z material and less likely with high incident energy
- Overall, Compton scattering is more dominant with higher incident energy in the same material
- But the percent of energy deposited due to photoelectric event is larger because all incident energy is absorbed.

Measures of X-ray Beam: Photon Count

- Photon fluence:

$$\Phi = \frac{N}{A}$$

- Photon fluence rate:

$$\phi = \frac{N}{A\Delta t}$$

- N: number of photons
- A: area

Measures of X-ray Beam: Energy Flow

- Energy fluence:

$$\Psi = \frac{N\hbar\nu}{A}$$

- Energy fluence rate:

$$\psi = \frac{N\hbar\nu}{A\Delta t}$$

- Intensity: $(= \psi)$

$$I(E) = \frac{NE}{A\Delta t}$$

Spectrum of X-Ray

- The x-ray beam produced by an x-ray tube (mainly Bremsstrahlung) is polyenergetic (consisting photons with different energies or frequencies)
- X-ray spectrum S(E):
  - The number of photons with energy E per unit area per unit time

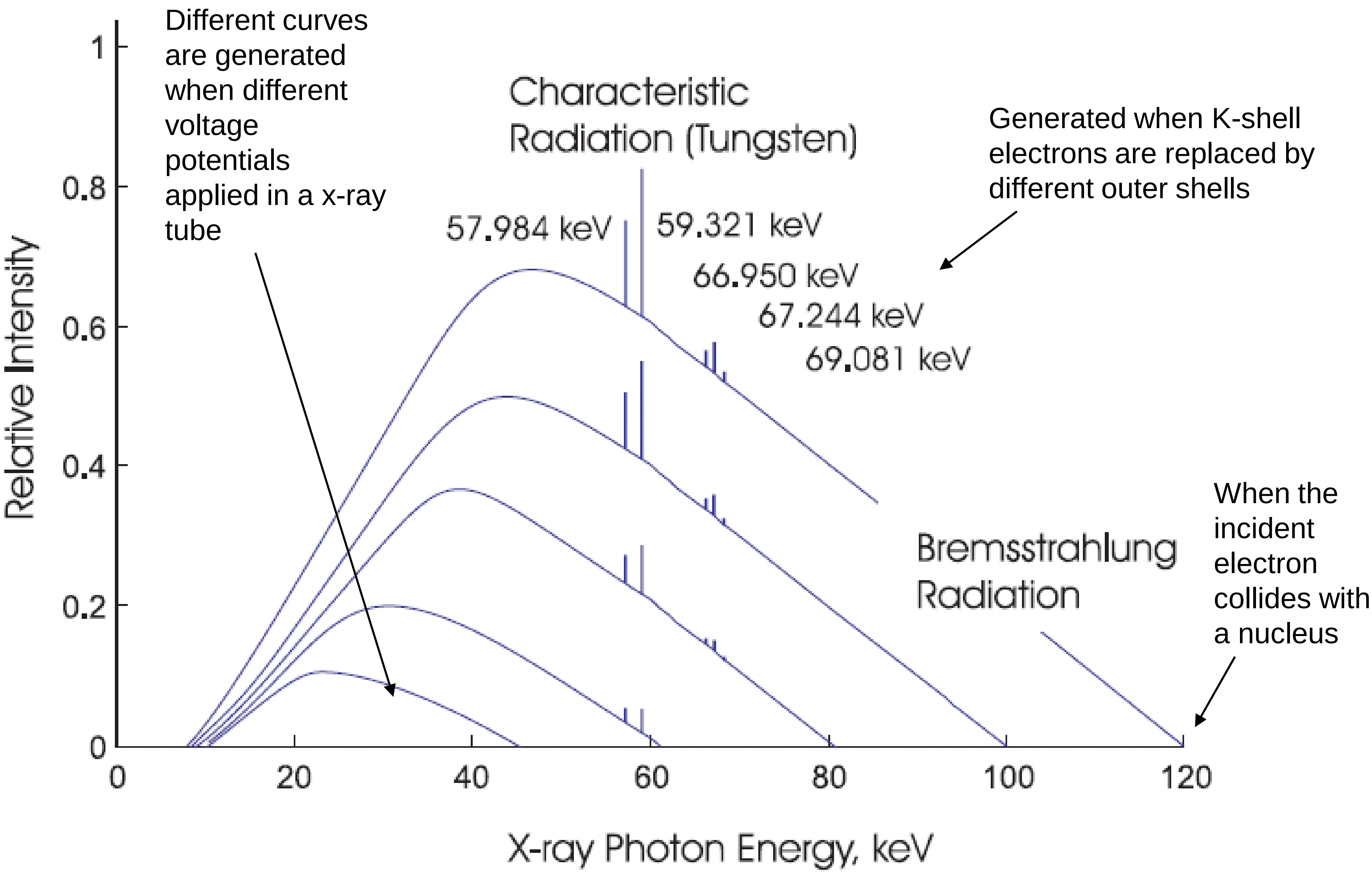
- Photon fluence rate from spectrum:

$$\phi = \int_0^\infty S(E')\,dE'$$

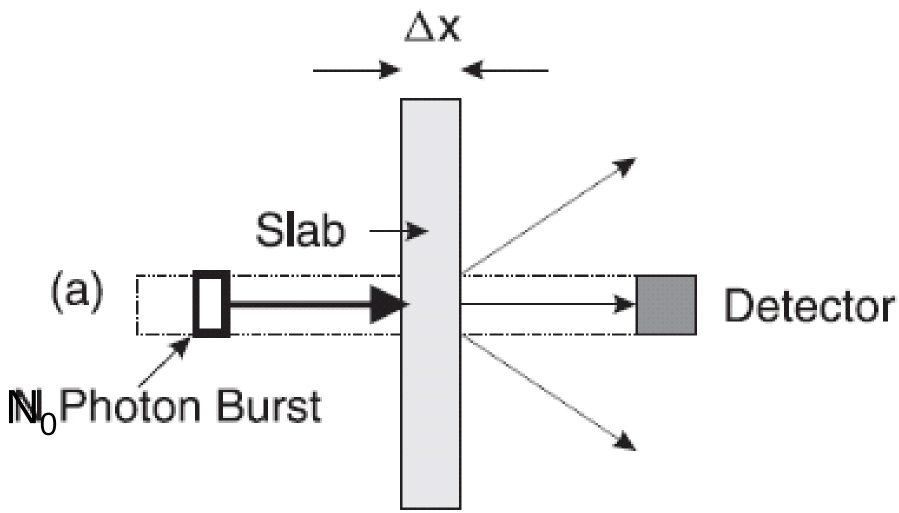
- Intensity from spectrum:

$$I = \int_0^\infty E' S(E')\,dE'$$

Spectrum of X-Ray



Attenuation of X-ray Radiation: Homogeneous Slab



Photons will be absorbed/deflected through the slab
Let # of photons at x = N(x)
photons lost from position x to x+dx can be approximated by
$dN=N(x+dx)-N(x) = -\mu N(x)dx$, when dx is very small
linear attenuation coefficient: μ
μ is the fraction of photons that are lost per unit length
The above can be rewritten as
$dN(x)/dx = -\mu N(x)$. Integrate this from x=0 to Δx yields
$N(x) = N_0 \exp\{-\mu x\}$ ← The fundamental photon attenuation law

Linear Attenuation Coefficients of Biological Tissues

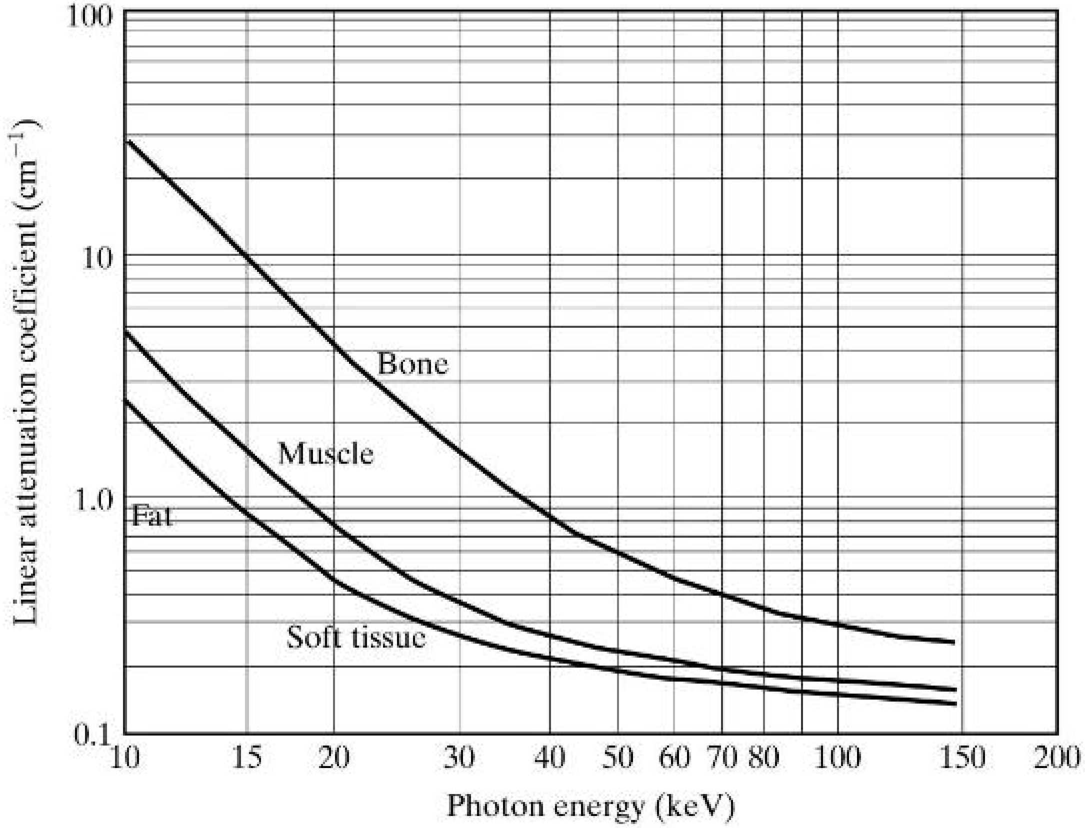


Figure 4.8

Medical Imaging Signals and Systems, by Jerry L. Prince and Jonathan Links.

Homogeneous Slab

- Homogeneous slab: the attenuation rate is the same over the entire slab

- Homogeneous slab thickness Δx
- Fundamental photon attenuation law

$$N = N_0 e^{-\mu \Delta x}$$

- μ is linear attenuation coefficient
- In terms of intensity:

$$I = I_0 e^{-\mu \Delta x}$$

This is known as Beer's Law

Half-Value Layer (HVL)

- Homogeneous slab (shielding)
- HVL = thickness that will stop half the photons

$$\frac{1}{2} = \exp\{-\mu \, \mathrm{HVL}\}$$

- Relation to μ

$$\mathrm{HVL} = \frac{0.693}{\mu}$$

Example

- Consider the image taken of a bar phantom uniformly irradiated by monoenergetic x-ray photons
  – Assuming the bars are made of material that has a HVL of 0.2cm, and bars have thickness of 0.4 cm
  – Assuming x-ray photons pass through the space between bars w/o attenuation
  – Assuming the intensity of the image is proportional to the number of detected photons in a unit area
  – What is the contrast of the resulting image?

- Go through in the class

Non-Homogeneous Slab

- The attenuation coefficient depends on x

  - Non-homogeneous slab:

$$\frac{dN}{N} = -\mu(x)dx$$

  - Integration yields:

$$N(x) = N_0 \exp\{-\int_0^x \mu(x')dx'\}$$

  - For intensity:

$$I(x) = I_0 \exp\{-\int_0^x \mu(x')dx'\}$$

Example: Two Layer Slab

- A slab with two homogeneous layers, with thickness d1, d2, and attenuation coefficients μ1, μ2. If the input X-ray intensity is N0, what is the intensity at the other end of the slab?

Polyenergetic Photons

- The linear attenuation coefficient depends on the medium property as well as the energy of the incident photon (E)
- For a given material, μ can be denoted by μ(x;E)
- When the incident photons are polyenergetic, with spectrum S(E), the outgoing photon spectrum is

$$S(x;E) = S_0(E)\exp\left\{-\int_0^x \mu(x';E)dx'\right\}$$

- In terms of intensity

$$I = \int_0^\infty E' S(E')dE'$$

$$I(x) = \int_0^\infty S_0(E')E'\exp\left\{-\int_0^x \mu(x';E')dx'\right\}dE'$$

Radiation Dosimetry

- Previous topics deal with the production of radiation and measurement of radiation wave
- Radiation dosimetry considers the effect of radiation to the interacting media
  - Exposure
  - Dose
  - Kerma
  - Effective dose

Exposure (Creation of Ions)

- Exposure (X) is measured in terms of the number of ions produced in a specific volume of air by EM radiation
- SI unit: C/kg
- Common unit: Roentgen (R)
  - 1 C/kg = 3876 R
  - Wilhelm Roentgen: Discovered X-ray, 1895, Nobel Prize 1901
- Exposure decreases with distance from source (d) following a inverse square law

$$X(d) = X(0)/d^2$$

Dose (the deposition of energy)

- How much energy is deposited into material?
- Dose, D, the energy deposited per unit volume
- SI unit: Gray (Gy) $1\ \text{Gy} = 1\ \text{J/kg}$
- Common unit: rad

$$1\ \text{Gy} = 100\ \text{rads}$$

- When $X = 1$ R soft tissue incurs 1 rad absorbed dose.

Kerma

- How much energy is deposited into the *electrons*?
- Kerma, K, is the energy deposited into the electrons of a material
- SI units: Gray (Gy) = 1 J/kg = 100 rads
- At diagnostic energies in the body, $K = D$
- (In general, $K \geq D$. Some electrons can cause bremsstrahlung and their energy irradiated away $\rightarrow$ no dose. Not likely in body.)

Dose vs. Exposure

$$D = fX$$

f - factor depends on material:

$$f = 0.87 \frac{\left(\mu/\rho\right)_{material}}{\left(\mu/\rho\right)_{air}}$$

$\left(\mu/\rho\right)$: mass attenuation coefficient

$f = 0.87$ for air

$f \approx 1$ for soft-tissue

See Table 4.6 for the mass attenuation coefficient of typical materials

Equivalent and Effective Dose

- Dose equivalent
  - The effect of radiation depends on the source of radiation (energy)
  - Dose equivalent: $H = D\,Q$
  - Q: quality factor
    - $Q = 1$ for x-ray, gamma ray, electron, beta particle (used in medical imaging)
    - $Q = 10$ for neutrons and protons
    - $Q = 20$ for alpha particles
- Effective dose
  - Effect of a dose also depends on the tissue type
  - Effective dose measures the average effect over different tissue types

$$D_{effective} = \sum_{organs} w_j H_j$$

$$w_j \text{ :weighting factor for organ } j$$

- D can be measured in rads, H can be measured in rems
- For a dose of 1 Gy and Q =1 -> H = 1 sievert (Sv)

Table 2.2: Tissue weighting factors for calculations of effective dose

Tissue / organ	Tissue weighting factor, w_T
Gonads	0.2
Bone marrow (red)	0.12
Colon	0.12
Lung	0.12
Stomach	0.12
Chest	0.05
Bladder	0.05
Liver	0.05
Thyroid	0.05
Oesophagus	0.05
Average (brain, small intestines, adrenals, kidney, pancreas, muscle, spleen, thymus, uterus)	0.05
Skin	0.01
Bone surface	0.01

Effective Dose of Different Tissues [Smith&Webb]

Table 2.3: Radiation doses from common planar radiography and CT scans

Procedure	Effective dose (mSv)
Abdominal planar X-ray	1.5
Chest planar X-ray	0.04
Lumbar spine planar X-ray	2.4
Intravenous pyelogram	4.6
Chest CT scan	8.3
Brain CT scan	1.8
Abdominal CT scan	7.2

Summary

- Ionization: ejection of an orbiting electron from an atom, the affected atom produces radiation in the process of returning to ground state
- Two types of ionizing radiation
  - Particulate
  - EM
- Particulate radiation transfers energy via
  - Collisional transfer: resulting in heat
  - Radioactive transfer: resulting in characteristic x-ray and Bremsstrahlung x-ray
  - X-ray is produced by energetic electrons accelerated in a x-ray tube, consisting primarily Bremsstrahlung x-ray
- EM radiation transfers energy via
  - Photoelectric effect: incident photons completely absorbed
  - Compton scattering: incident photons are deflected
  - X-ray imaging utilizes EM photoelectric effect caused by injected X-ray photons.

Summary (cntd)

- Attenuation of EM radiation:
  - Linear attenuation coefficient is the fraction of photons that are lost per unit length
    - Depends on material property and the incident photon energy
  - Fundamental photon attenuation law
    - Homogeneous slab $N = N_0 e^{-\mu \Delta x}$
    - Heterogeneous slab $N(x) = N_0 \exp\{- \int_0^x \mu(x') dx'\}$

- Radiation dosimetry
  - Exposure vs. dose: D=fX
  - Equivalent dose: H=DQ
  - Effective dose:

$$D_{effective} = \sum_{organs} w_j H_j$$

w_j :weighting factor for organ j

Reference

- Prince and Links, Medical Imaging Signals and Systems, Chap 4.

Homework

- Reading:
  - Prince and Links, Medical Imaging Signals and Systems, Chap 4.
- Will be assigned at the end of lecture 3 and due on Friday Sept 29