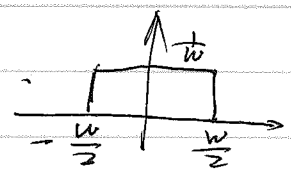
Medical Imaging, Fall 2011. Midterm solutions

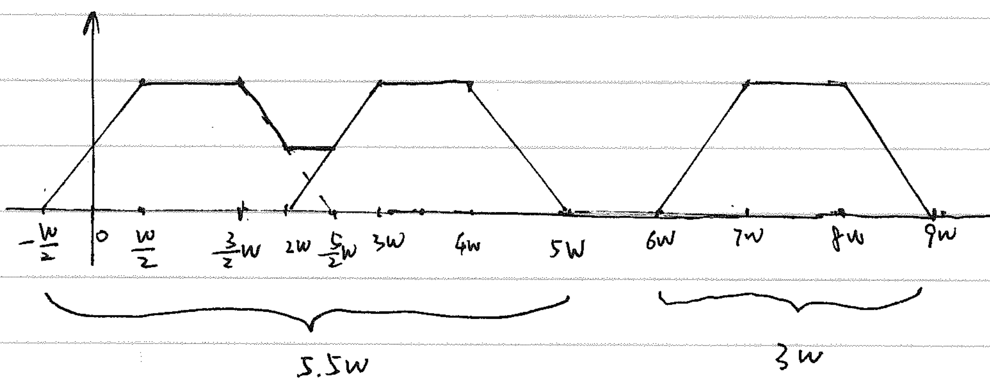
1\. (a) $$\ell(x) = \begin{cases} \frac{1}{w} & |x| < \frac{w}{2} \\ 0 & \text{otherwise} \end{cases}$$



$$\text{FWHM} = \frac{w}{2} \times 2 = w$$

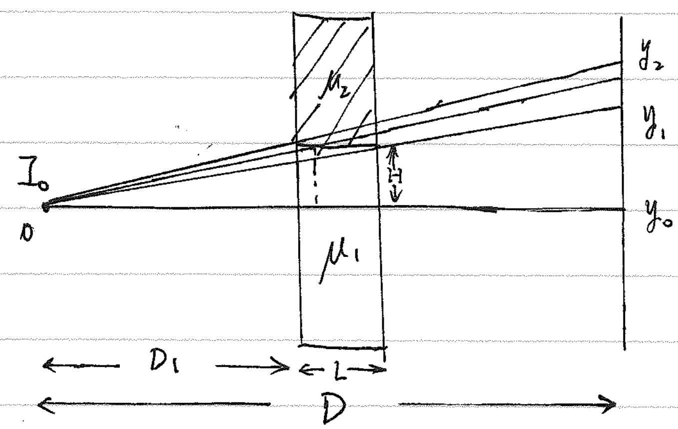
$$\text{resolution} = \frac{1}{\text{FWHM}} = \frac{1}{w}$$

(b)



First two bars can not be told apart, while the third one can. So there are two bars with width 5.5w and 3w, respectively.

2.



Case 1, when the imaging point is below y_1, the X-ray only go through slab with attenuation coefficient μ_1.

$$I(\theta) = \frac{I_0}{4\pi D^2} \cos^3\theta \, e^{-\mu_1 L/\cos\theta}, \quad \theta \in \left(-\frac{\pi}{2}, \tan^{-1}\left(\frac{H}{D_1+L}\right)\right)$$

Case 2, when the imaging point is above y_2, the X-ray only go through slab with attenuation coefficient μ_2.

$$I(\theta) = \frac{I_0}{4\pi D^2} \cos^3\theta \, e^{-\mu_2 L/\cos\theta}, \quad \theta \in \left[\tan^{-1}\left(\frac{H}{D_1}\right), \frac{\pi}{2}\right)$$

Case 3, when the imaging point is between y_1 and y_2, the X-ray go through μ_1 and μ_2. The path length in μ_1 and μ_2 depends on θ.

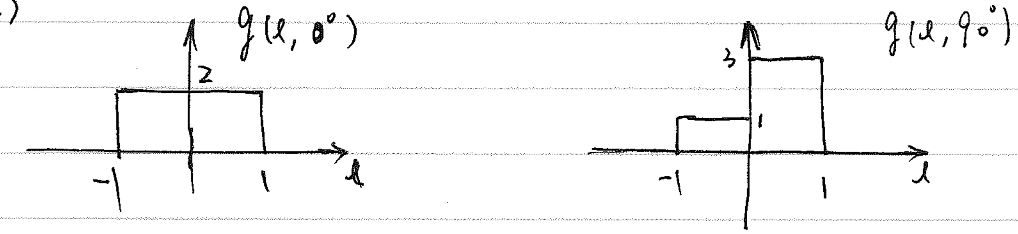
The path length in μ_1 is $\frac{H}{\tan\theta} - D_1$,

— — — μ_2 is $L - \left(\frac{H}{\tan\theta} - D_1\right)$.

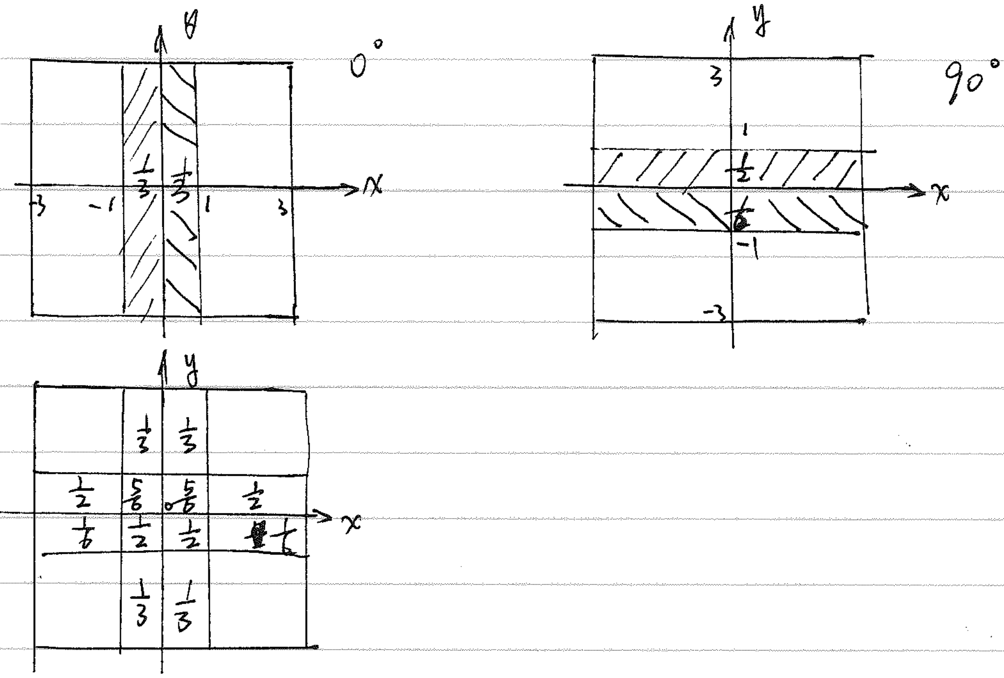
$$I(\theta) = \frac{I_0}{4\pi D^2} \cos^3\theta \, e^{-\left[\mu_1\left(\frac{H}{\tan\theta} - D_1\right) + \mu_2\left(L - \left(\frac{H}{\tan\theta} - D_1\right)\right)\right]/\cos\theta}$$

$$\theta \in \left[\tan^{-1}\left(\frac{H}{D_1+L}\right), \tan^{-1}\left(\frac{H}{D_1}\right)\right)$$

3. (a)



(b)



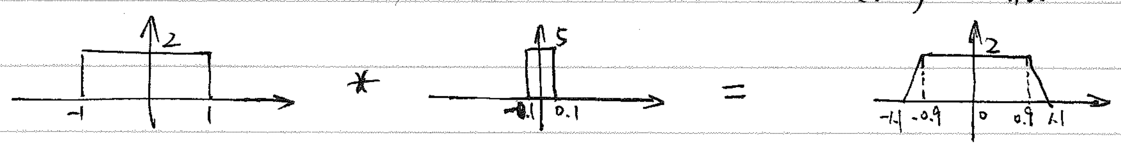
(c) Based on projection - slice theorem

$$F(\rho\cos\theta, \rho\sin\theta) = \mathcal{F}_{1D}(g(l,\theta))$$

when $\theta = 0^\circ$, $\mathcal{F}_{1D}(g(l,0)) = \mathcal{F}\{2\,\mathrm{rect}(\frac{x}{2})\}$

$$= 4\,\mathrm{sinc}(2u) = \frac{2}{\pi u}\sin(2\pi u)$$

(d)



(e) $\hat{f}(x,y) = f(x,y) * h(r)$, $r = \sqrt{x^2+y^2}$

Suppose the detection response function is : $s(l) = \mathrm{rect}(\frac{l}{d}) \Rightarrow S(\rho) = d\,\mathrm{sinc}(d\rho)$

$$h(r) = \mathcal{H}^{-1}\{d\,\mathrm{sinc}(d\rho)\} , \quad d = 0.2\text{ cm}$$

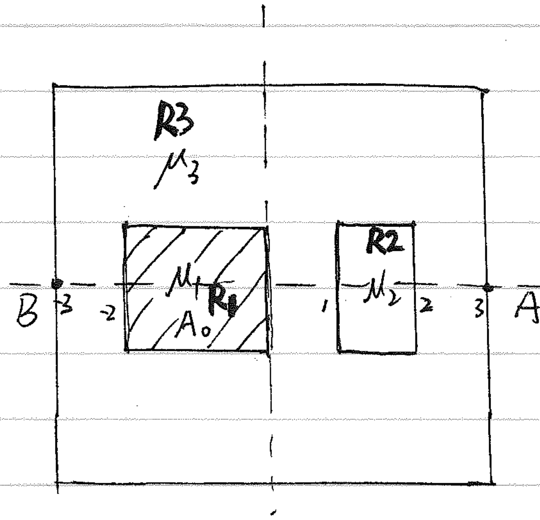
4. (a) The radiotracer for SPECT should has radioactive atoms whose decay produces a single gamma photon directly.

(b) The radiotracer for PET should be the radioactive atoms whose decay produces a positron that is subsequently annihilated, producing two gamma photons.

(c) SPECT need a collimator, while PET does not. PET use annihilation coincidence detection (ACD) to reject gamma rays that are not generated by ~~can~~ a single positron. Since ACD provide information about the direction of travel of the photons, collimator is not required with this method. and is in fact undesirable, because it reduces the sensitivity of the detectors.

(d) The desirable range of the half-life of the radiotracer for medical imaging should be on the order of minutes to hours, about the time it takes ~~to~~ perform study.

6. (a)



Note that radioactivity only exists in R1 !

$$t_{\frac{1}{2}} = T = \frac{0.693}{\lambda} \Rightarrow \lambda = \frac{0.693}{T}$$

$$A_t = A_0 e^{-\lambda t} = A_0 e^{-\frac{0.693}{T}t}$$

$$\Phi_A = \int_{-2}^{0} \frac{A_t}{4\pi(3-s)^2} \exp\{-\mu_1(-s)-\mu_2-2\mu_3\}\, ds$$
$$= \int_{-2}^{0} \frac{A_0 e^{-\frac{0.693}{T}t}}{4\pi(3-s)^2} \exp\{s-8\}\, ds$$

$$\Phi_B = \int_{-2}^{0} \frac{A_t}{4\pi(s+3)^2} \exp\{-\mu_1(s+2)-\mu_3\}\, ds$$
$$= \int_{-2}^{0} \frac{A_0 e^{-\frac{0.693}{T}t}}{4\pi(s+3)^2} \exp\{-s-5\}\, ds$$

(b) $$\Phi_{AB} = \int_{-2}^{0} A_t\, ds \cdot \exp\{-2\mu_1-\mu_2-3\mu_3\}$$
$$= 2A_t \cdot \exp\{-13\}$$
$$= 2A_0 e^{-\frac{0.693}{T}t} \exp\{-13\}$$