

EL5823/BE6203/G16.4426 Medical Imaging, Spring 2014

Midterm Exam, 3/24/2014, 10:20-12:50 PM

(closed book, 1 sheet of notes double sided allowed)

(No electronics allowed. Please leave all electronics in your backpack and leave your backpack in the front of the classroom. Cheating will result in an F grade)

1. (15 pt) Suppose an imaging system has an impulse response that is separable so that $h(x, y) = h_x(x)h_y(y)$ and that $h_x(x) = h_y(x) = l(x)$, with $l(x)$ being a short duration rectangular function with width w as illustrated in Fig. 1(a). (a) (5pt) what is the minimal horizontal and vertical, respectively, distances between two objects so that the two objects can be seen separated by the imaging system? (b) (10 pt) Suppose a slice contains 6 objects as shown in Fig. 1(b), sketch the image obtained by the imaging system. Was the imaging system able to tell all objects apart? By sketching, I mean that you should draw roughly the boundaries of the imaged objects.

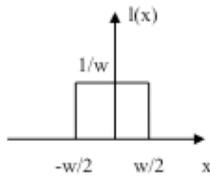


Fig. 1(a)

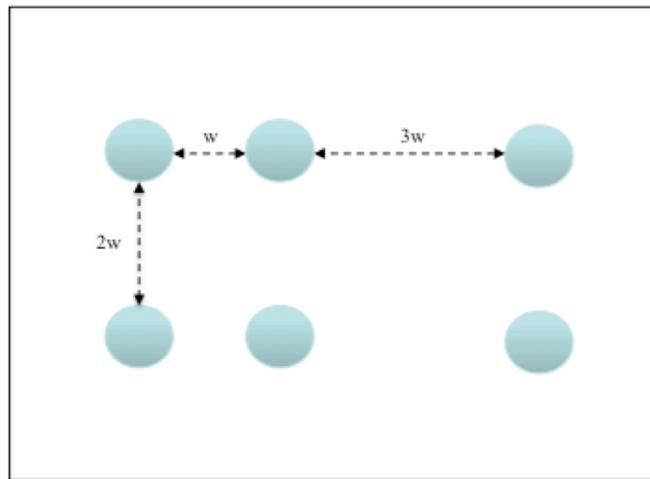


Fig. 1(b)

2. (10 pt) Consider an X-ray tube. (a) If the voltage applied between the anode and the cathode is 100 KV, and the binding energy of K-shell electrons in the anode is 50 KeV, and that of L-shell and M-shell are respectively 8 KeV and 2 KeV. Sketch the shape of the resulting x-ray energy spectrum. Properly label your horizontal axis. You only need to consider characteristic rays generated when ejected K-shell electrons are replaced by L-shell and M-shell electrons. (b) Sketch the energy spectrum created when the voltage is changed to 40 KV, on the same figure you generated for part (a). Would there be characteristic rays in this case? Why? (c) What are the functions of contrast agents in some X-ray imaging?
3. (10 pt) Answer following questions briefly: (a) In X-ray CT, where is the x-ray source located (inside or outside the body)? what is measured at the detector? What properties of the tissue does the reconstructed image reveal? (b) answer the same set of questions for nuclear imaging (SPECT or PET).
4. (10 pt) (a) In nuclear imaging, what type of radiotracer is needed for SPECT? (b) What type is needed for PET? (c) Do we need to use a collimator in SPECT camera? What about for PET? Why? (d) How does the half-life time of the radiotracer affect the timing for a nuclear imaging test? Why?

5. (15 pt) Consider the x-ray imaging of a medium illustrated in Fig. 5. (a) (10pt) Assume the x-ray source is an ideal point source with intensity I_0 , as illustrated in the figure below. Determine the intensity of detected photons along the y axis on the detector plane. Express your solution in terms of the y -coordinate. Sketch this function. You should consider the inverse square law and the oblique effect. (b) (5pt) How will the detected signal look if the source is uniformly distributed over a small disk of diameter D ? Sketch the signal in the y -direction. How is the resulting signal mathematically related to the solution for part (a)? What is the equivalent filter's impulse response? For this part, assume the object thickness L is close to 0.

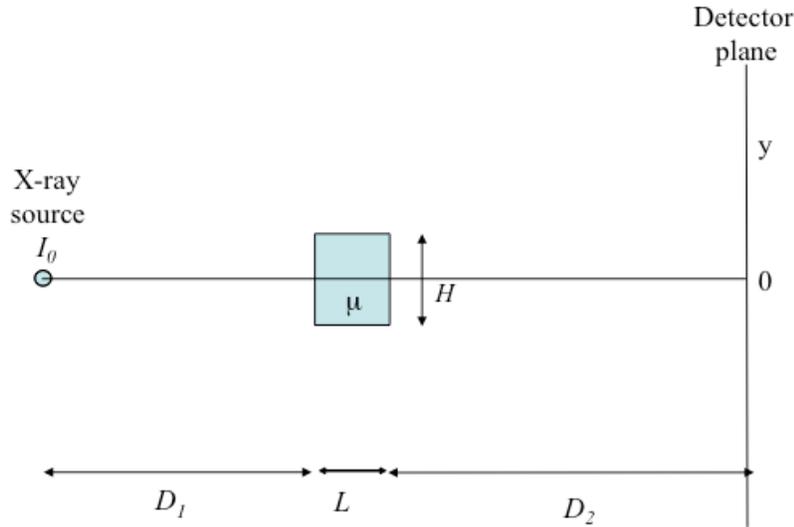


Fig. 5

6. (20 pt) Suppose the tissue slice (with dimension 6x6cm) being imaged by a parallel beam x-ray CT scanner contains distribution of the linear attenuation coefficients as shown in Fig. 6, with $\mu_1 = 1, \mu_2 = 2, \mu_3 = 3 \text{ cm}^{-1}$ (a) (4pt) Assume the detector is a point detector. Sketch the projection $g(l, \theta)$ as a function of l , for $\theta=0$ (detector on the bottom) and 90 degrees (detector on the right), respectively. You should indicate the magnitudes of the projected values where necessary on your sketch. Also clearly specify any transition points in the l -axis. (b) (4pt) Sketch the image obtained by backprojections from both 0 and 90 degree projections. You should assume that you know the dimension of the tissue being imaged and normalize your backprojection using the known dimensions. (c)(2pt) As you can see, the backprojected image was not able to correctly identify all the objects in the slice. For this particular case, if you could collect data in one more projection angle, which projection angle would provide the most gain in revealing the actual object locations, and why? (d) (4pt) Determine the Fourier transform of the original image along a line with orientation $\theta=0$ degree in the frequency domain. (e) (4pt) What will be the projected function for $\theta=0$ if the detector is an area detector with width 0.2 cm. Sketch the projection function. (f)(2pt) How is the image recovered from the readings obtained with such a detector related to the image recovered from using an ideal point detector? (assume that in both cases you use the convolution projection method with the same filter). (Note: for your solution, if you need to know the Hankel transform of a function, you can just state that it is the Hankel transform of that function, without giving the actual function form).

Hint: you could make use of the following Fourier transform pair and property:

$$g(x) = \begin{cases} 1, & \text{if } |x| < \Delta/2 \\ 0, & \text{otherwise} \end{cases} \leftrightarrow G(f) = \Delta \frac{\sin(\pi\Delta f)}{\pi\Delta f}$$

$$g(x - x_0) \leftrightarrow G(f)e^{-j2\pi f x_0}$$

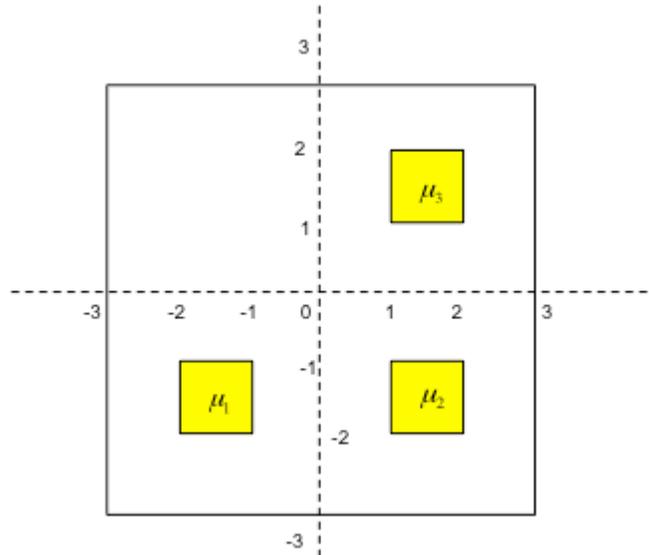


Fig. 6

7. (15 pt) A 2-D slice to be imaged is shown in Fig. 7, which consists of two regions with linear attenuation coefficients μ_1 and μ_2 . The background has a linear attenuation coefficient of μ_3
- (8 pt) Suppose a solution containing a gamma ray emitting radio tracer with an initial radioactivity of A_1 and half life time of T_1 is injected into both regions. We image the radioactivity distribution in this slice using a rotating SPECT camera. Compute the measured signal by the camera at positions A and B, respectively at time $t = T_1 / 2$ after the injection of the radionuclide solution. Note: you can just express you solutions in terms of properly written integrals.
 - (5 pt) Now suppose the radio tracer in (a) is replaced by a positron emitting radio tracer with the initial radioactivity A_2 and half-life T_2 . This time the slice is imaged using a PET scanner. Compute the measured signal by the pair of cameras positioned at A and B, at time $t = T_2 / 2$ after the injection of the radionuclide solution.
 - (2pt) How is the measured signal at time $t = T_1$ related to that measured at time $t = T_1 / 2$ in SPECT ? What about PET?

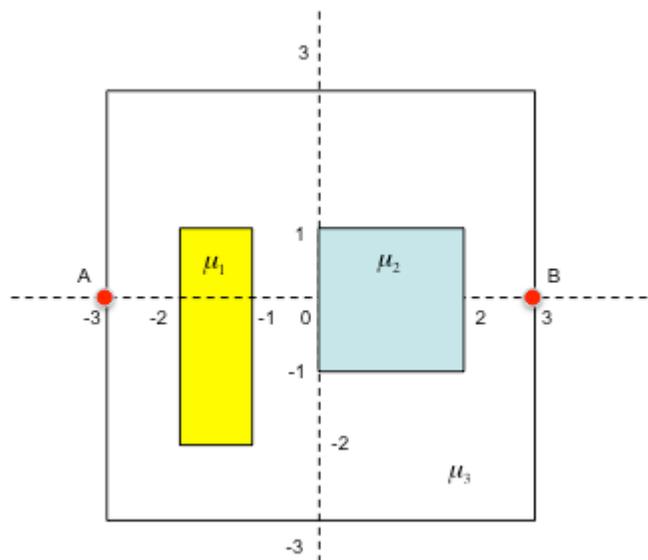


Fig. 7