EL5823/BE6203/G16.4426 Medical Imaging, Spring 2013

Midterm Exam, 3/25/2013, 3:00-5:30PM
(closed book, 1 sheet of notes double sided allowed)

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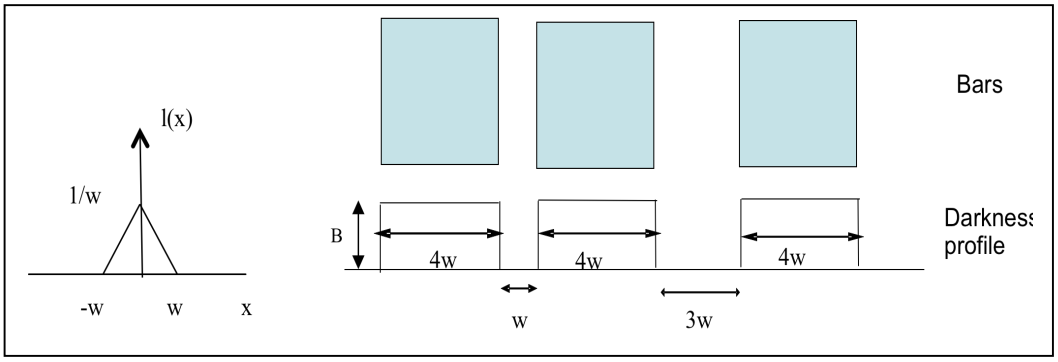
1. (10 pt) The line spread function of an imaging system is described by a triangular box function where the unit of *W* is millimeter (mm), as illustrated in the figure below on the left. What is the resolution of this imaging system in terms of FWHM (full width at half maximum) and in terms of lines/mm? (b) Suppose the field to be imaged contains 3 parallel bars of width *4W* mm, spread non-uniformly, with the darkness profile as indicated in the figure below. Determine the darkness profile of the bars after imaging. Can you still tell all the bars apart?



2. (10 pt) Assuming the probability density functions of a blood test result for patients with and without a disease are described by exponential distributions:

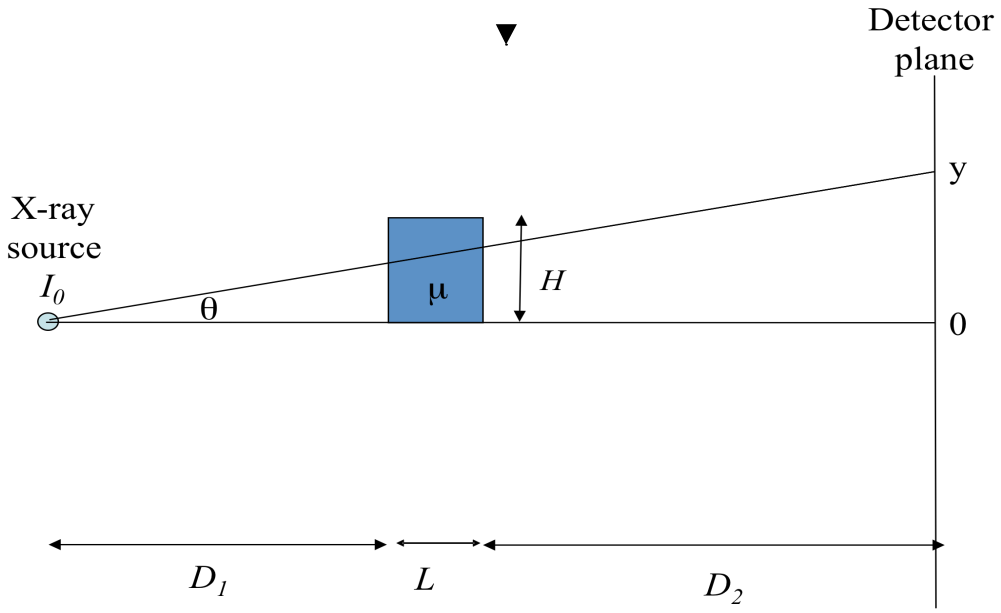
$$Normal: p_N(x) = \frac{a}{2}\exp\{-a|x-\mu_n|\}, \quad Diseased: p_D(x) = \frac{b}{2}\exp\{-b|x-\mu_d|\}, \mu_n < \mu_d$$

Assume the diagnosis is determined based on a threshold *t*. For a patient with test value below *t*, we call it normal. Otherwise, we call it diseased. (a) Determine the false positive probability (FPP, the probability that a patient is diagnosed to have a disease when he/she does not); (b) Determine the false negative probability (FNP, the probability when a patient is considered normal when he/she has the disease). (c) When you increase *t*, will the FPP increase or decrease? How about FNP? (d) One way to choose *t* is by minimizing a weighted combination of FPP and FNP, with weights *c* for FPP, and *d* for FNP. Do you think in general *c* should be greater or smaller than *d*? (e) What would be the optimal *t* using this criterion? Express all your solutions in terms of given parameters *a,b,c,d,* μ_n, μ_d.

Note: if you don't have enough time to complete the calculation, please make sure that you at least write down all the equations clearly.

3. (15 pt) (a) Describe briefly how X-ray is created in an X-ray tube, including the two primary form of radiation. (b) If the voltage applied between the anode and the cathode is 200 KV, and the binding energy of K-shell electrons in the tungsten anode is 70 KeV, and that of L-shell and M-shell are respectively 11 KeV and 3 KeV. Sketch the shape of the resulting x-ray energy spectrum. Properly label your horizontal axis. You only need to consider characteristic rays generated when ejected K-shell electrons are replaced by L-shell and an M-shell electrons. (c) Sketch the energy spectrum created when the voltage is changed to 60 KV, on the same figure you generated for part (b). Would there be characteristic rays in case (c)? Why? (d) Why are patients asked to drink a "chalky milkshake" containing barium when undergoing gastrointestinal tract imaging?

4. (15 pt) Consider the x-ray imaging of a medium containing a slab with a finite dimension in the y-direction, illustrated below. (Note that we assume the slab is infinitely long in the direction orthogonal to the plane of the drawing). (a) Assume the x-ray source is an ideal point source with intensity I_0, as illustrated in the figure below. Determine the intensity of detected photons along the *y* axis on the detector plane. Express your solution in terms of the y-coordinate. Sketch this function. You should consider the inverse square law and the oblique effect. (b) How will the detected signal look if the source is uniformly distributed over a small disk of diameter *D*? For this part, assume the slab is very thin, i.e. *L* is close to 0. Sketch the signal in the y-direction.



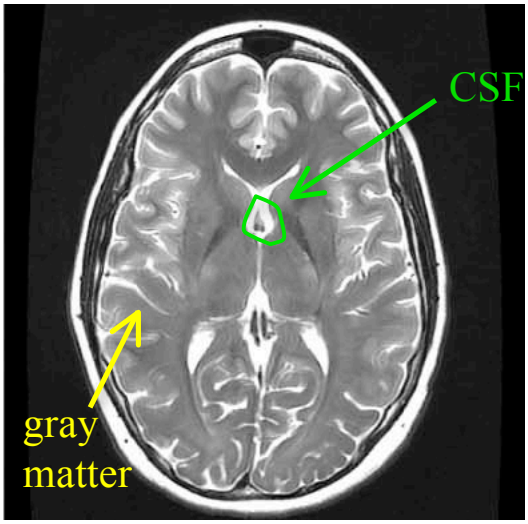
5. (20 pt) T2 relaxation causes a decay of the transverse component of magnetization ($M_\perp$) according to the equation:

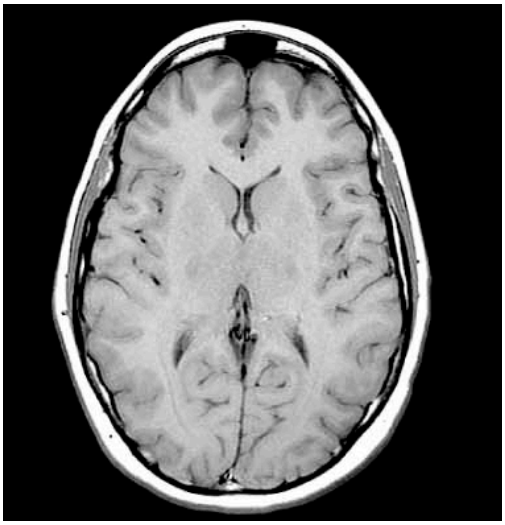
$$\frac{dM_\perp}{dt} = -\frac{M_\perp}{T_2}$$

a) If $M_\perp(0)$ is the initial value of the transverse magnetization, immediately after an RF excitation, what is the solution of the equation above?
b) What is the difference between T2 and T2* relaxation?
c) Plot $M_\perp(t)$ vs. t for both the cases of T2 and T2* relaxation. Marking values of $M_\perp(t)$ at $t = 0$ and $t = \infty$.
d) If you want an image with T2-weighted contrast, would you use a gradient echo (GRE) or spin-echo (SE) sequence? Why?

6. (10 pt) a) Complete the table below to show what choice of TE/TR correspond to T1-weighted, T2-weighted, or Proton-Density-weighted contrast (one of the choice is not useful), in the case of a spin-echo pulse sequence. Explain your answers. b) Given that gray matter has T2 ~ 77ms and T1 ~ 760 ms, whereas cerebral spinal fluid (CSF) has T2 ~ 280 ms and T1 ~ 2650 ms, indicate which of the following two images is T1-weighted and which is T2-weighted. Explain your answers.

	Long TR	Short TR
Short TE		
Long TE		





7. (10 pt) Match the essential components of an MRI system listed in the left column with one or more descriptions on the right column:

i) Magnet
ii) Gradient and shim coils
iii) RF surface coils

_____ A) excite the spins
_____ B) polarize the spins
_____ C) encode spatial information
_____ D) incorporated in the MR system
_____ E) detect emitted signal
_____ F) compensate for B_0 inhomogeneities

8. (5 pt) (a) Explain how slice selection work in MRI. (b) What imaging parameters can be adjusted to control the thickness of the selected slice?

9. (5 pt) (a) Describe the BOLD effect and how it is used in functional MRI (fMRI). (b) What factors limits the spatial resolution of fMRI maps? What source of error is associated with that?