Polytechnic Institute and School of Medicine, New York University
EL582/BE620/G16.4426 Medical Imaging, Spring 2012
(closed book, 2 sheets of notes double sided allowed)
5/14/2012, 3-5:50PM, RH602
With solution

Part 1 ---- Optical imaging (15 points total)

1. (2 pt) Which of the following components are used in Differential Interference Contrast microscope (DIC)? (Mark all that apply)
   a. Polarizer.
   b. Light source.
   c. Condenser.
   d. Objective.
   e. Wollaston prism.
   f. Pelin-Broka prism.
   g. All answers.

Answer: a,b,c,d,e

2. (2 pt) Which of the following are used in phase-contrast microscope ? (Mark all that apply)
   a. Optical fiber.
   b. Phase plate.
   c. Wollaston prism.
   d. Condenser Annulus.
   e. Light source.
   f. All answers.

Answer: b, d, e

3. (5 pt) The following is a list of fluorophores.
   - RFP (Red fluorescent protein).
   - Alexa647 Dye.
   - 655 Quantum-Dots.

   a. Which of the fluorophores below is the most photostable? Answer: 655 Quantum-Dots
   b. Which is the least photostable? Answer: RFP
   c. Which is the brightest? Answer: Quantum dots
   d. Which is the smallest? Answer: Alexa647 Dye
   e. Which is the largest? Answer: quantum dots

4. (2 pt) What are the main **advantages** of confocal microscopy over widefield microscopy? (Mark all that apply)
   a. Point scanning.
   b. Faster image acquisition.
   c. Sensitivity.
   d. Smaller excitation/imaging volume.

e. Complex instrumentations.
f. All answers.

Answer: a, d

5. (2 pt) What are the main **disadvantages** of TIRF microscopy compared to widefield microscopy? (Mark all that apply)
   a. Point scanning.
   b. Slower image acquisition.
   c. Imaging depth.
   d. Laser orientation.
   e. Sensitivity.
   f. All answers.

Answer: c

6. (2pt) Which of the following detectors are **not** used in TIRF microscopy? (Mark all that apply)
   a. Photodiode.
   b. CCD camera.
   c. Photomultiplier.

Answer: a, c

Part II ---- Ultrasound Imaging (35 points total, 5 bonus pts)

7. (10 pt) For each of the properties of an ultrasound transducer listed in the left column, choose the factor listed in the right column that is most important for determining the property:

i) Frequency	_____	A) Pulse length
ii) Axial resolution	_____	B) Matching layer
iii) Lateral resolution	_____	C) Thickness of piezoelectric material
iv) Depth of focus	_____	D) wavelength x (f-number)
v) Bandwidth	_____	E) wavelength x (f-number)^2

[Recall that f-number = focal length / aperture size of the transducer]

Answer:

i) Frequency	**C**	A) Pulse length
ii) Axial resolution	**A**	B) Matching layer
iii) Lateral resolution	**D**	C) Thickness of piezoelectric material
iv) Depth of focus	**E**	D) wavelength x (f-number)
v) Bandwidth	**B**	E) wavelength x (f-number)^2

8. (20 pt) You are given the task of imaging a human kidney with ultrasound.

a) (5 pt) For kidney imaging, the required penetration depth (distance below skin / transducer) is 8-cm. What is the maximum frame rate (images per second) for a 256-line image?
[assume the sound speed, c = 1540 m/s]

Propagation length = 8x2 = 16 cm = 0.16 m
Time per line = 0.16 / 1540 ~ 104 μs
Time per image = 104 x 256 ~26,600 μs = 26.6 ms
Frame rate = 1/26.6 x 10^{-3} s ~ 37 images per second

b) (5 pt) For imaging the kidney, choose between the following frequencies and justify your choice: 1-MHz, 5-MHz, 50-MHz.

5-MHz provides good resolution for this depth. 1-MHz leads to inferior resolution. 50-MHz has insufficient depth of penetration.

Note that: following answer to part c), at 50MHz, the attenuation at 8cm would be 2x8x50=800dB. The signal would be immeasurable.

c) (5 pt) For the transducer you chose in **b)**, estimate the signal loss due to an object at a depth of 4 cm below the skin. Estimate the expected signal amplitude, expressed as a percentage of the transmitted signal.

[Assume that attenuation = 1dB/cm/MHz, and recall that 20-dB is a 10x loss in signal amplitude.]

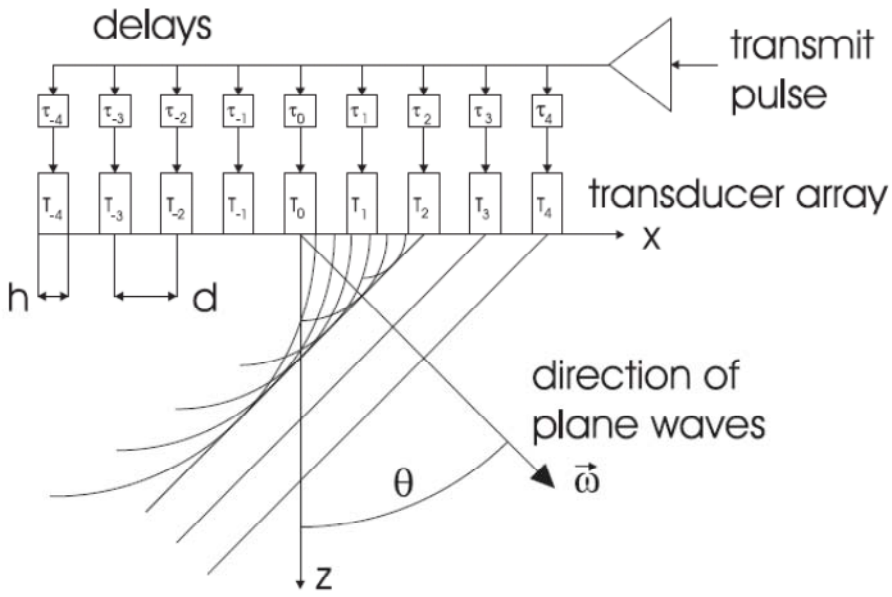
Depth 4-cm: Propagation length = 4 x 2 = 8-cm
Attenuation = 8 x 5 = 40 dB, or 100x signal loss, i.e., 1% of transmitted signal

d) (5 pt) As you scan the kidney, you notice that the renal artery is in the same direction as the propagation direction of ultrasound from your transducer (i.e., $\theta=0$) and that the maximum Doppler shift detected is 10-kHz. Estimate the peak blood velocity within the artery. [Recall the Doppler equation is $f_d = 2f_0\, v \cos\theta / c$, and assume the speed of sound in blood is 1600 m/s]

$\theta=0$, $\cos\theta=1$,
From Doppler equation:

$$v = cf_d/2f_0$$
$$= (1600 \text{ m/s}) \times (10 \times 10^3 \text{ /s}) / (2 \times 5 \times 10^6 \text{ /s})$$
$$= 1.6 \text{ m/s}$$

9. (5 pt, plus bonus) Consider a linear array transducer shown below:



a) (5 pt) To steer a beam in the direction θ, what is the time delay required between adjacent transducer elements (e.g., the delay between T_0 and T_1)?

Time delay = dsinθ/c (from geometry)

b) (Bonus Points, 5 pt) If the beam is steered in a direction θ_0, then it can be shown that the maximum amplitude of the main lobe occurs in the desired steering direction ($\theta=\theta_0$), but there are also unwanted "grating" lobes in directions θ_g, given by:

$$\sin\theta_g = \sin\theta_0 + i\lambda/d, \text{ for } i = 1,2,\ldots$$

For example, if $\theta_0 = 30°$ and $d = 2\lambda$, then a grating lobe occurs at $\theta_g = 90°$. For the same steering direction ($\theta_0=30°$), an array with $d = 4\lambda$ has grating lobes at
$\theta_g = 48.5°$ (i=1) and 90°(i=2).

Could there be a grating lobe if $d = \lambda$ (and $\theta_0 = 30°$)?

NO -

If $d = \lambda$, then $\sin\theta_g = 0.5 + 1 = 1.5$, which cannot be satisfied by any θ_g

How would the direction of the grating lobe change if $\theta_0 = -30°$ (still keeping $d = \lambda$)?

If $\theta_0 = -30°$, then

$\sin\theta_g = -0.5 + i$, so a grating lobe will occur at +30° (for i=1)

Is there a way to avoid ALL grating lobes, for any arbitrary steering direction?
(Hint: think about choosing d as a function of λ)

YES -

If $d < \lambda/2$, then $\sin\theta_g > 1$ for all steering angles θ_0 (positive and negative), which cannot be satisfied by any θ_g

Part III ---- MR Imaging (50 points total)

10. (15 pt)

a. (5pt) T1 relaxation, also known as "longitudinal relaxation", causes recovery of the longitudinal component of magnetization (M_z) to its equilibrium value M_0. The recovery process can be described by the following differential equation:

$$\frac{dM_z}{dt} = -\frac{M_z - M_0}{T_1}$$

Show that the solution to the equation above can be written as follows. Furthermore, sketch $M_z(t)$ as a function of t, marking values of $M_z(t)$ at $t = 0$ and $t = \infty$. Sketch the function for two different values of T_1 (one large, and one small) to show how T_1 influences the change of $M_z(t)$.

$$M_z = M_0 + \left[M_z(0) - M_0\right]e^{-t/T_1}$$

b. (2pt) The fact that T1 changes for different tissues can be exploited as a contrast mechanism in T1-weighted images. Would a tissue with long T1 appear dark or bright in such type of images? Why?

Dark. For T1-weighted images TR is not very long, so only the longitudinal magnetization of tissues with short T1 will have time to recover to its initial value M_0, whereas the signal of tissues with long T1 will be saturated after few repetitions (i.e. few TRs).

c. (8pt) Complete the table below to show what choice of TE/TR correspond to T1-weighted, T2-weighted, or Proton-Density-weighted contrast (one of the choice is not useful), in the case of a spin-echo pulse sequence. Explain your answers.

	Long TR	Short TR
Short TE		
Long TE		

Solution

	Long TR	Short TR
Short TE	**1) Proton-Density**	**2) T1-weighted**
Long TE	**3) T2-weighted**	**4) Not useful**

1) Short TE → minimize dephasing; long TR → no signal saturation. The combination of the two results in maximum signal from all tissues

2) Short TE → minimize dephasing; short TR → signal of tissues with long enough T1 will be saturated. The combination of the two allows creating a contrast that depends on the T1 of tissues.

3) Long TE → spins of tissues with long enough T2 will diphase (i.e. no signal); long TR → no signal saturation. The combination of the two allows creating a contrast that depends on the T2 of tissues (or T2* for gradient-echo sequences)

4) Long TE → spins of tissues with long enough T2 will diphase (i.e. no signal); short TR → signal of tissues with long enough T1 will be saturated. The combination of the two is not useful because the contrast will be a mix of T1 and T2 effects.

11. (15 pt)
    a. (9 pt) Magnetic resonance (MR) images are produced by applying gradients to the main magnetic field in order to spatially encode the signal. Explain how the slice selection, frequency encoding and phase encoding steps work.

Solution

By introducing magnetic field gradients, the frequency could be used to identify the position of the spins. Magnetic field gradients alter the precession frequency of the spins in a spatially-dependent manner.

Slice selection:

A magnetic field gradient is applied during the RF excitation pulse. The gradient alters the Larmor frequency ω_L of the spins along the direction of the gradient. Only those spins whose Larmor frequency equals the frequency of the RF pulse $\omega_L = \omega_{RF}$ will be excited. Such spins lie in a 'slice' of tissue perpendicular to the gradient

Frequency encoding:

A magnetic field gradient is applied during the data acquisition. The gradient alters the Larmor frequency ω_L of the spins in a spatially-dependent manner. The frequency of the signal emitted by each spin will therefore depend on its location along the direction of the gradient. The frequency thus provides a 'label' to identify the spins' location

Phase encoding:

A magnetic field gradient is applied in the remaining direction for a short period after excitation but before data acquisition. The gradient imparts a spatially-varying phase shift to the spins. During the subsequent data acquisition period, the spins along any line in the phase-encoding direction will precess with identical frequencies but different phases.

b. (2 pt) In MRI, we acquire the signal in k-space. Write an expression that relates the measured k-space signal $s(k_x,k_y)$ with the image signal S(x,y).

$$s(k_x,k_y)=\int S(x,y)\exp\left[-i2\pi(k_x x+k_y y)\right]\mathrm{d}x\mathrm{d}y$$

c. (2pt) Explain a method that we can use to recover S(x,y) from $s(k_x,k_y)$.

Once we have sampled sufficient data in k-space, we can perform a 2D inverse Fourier transform to recover the signal distribution in image space:

$$S(x,y)=\int s(k_x,k_y)\exp\left[i2\pi(k_x x+k_y y)\right]\mathrm{d}k_x\mathrm{d}k_y$$

d. (2pt) How would the image look if we sample only the center of k-space and fill with zeros the rest of the signal matrix? Why?

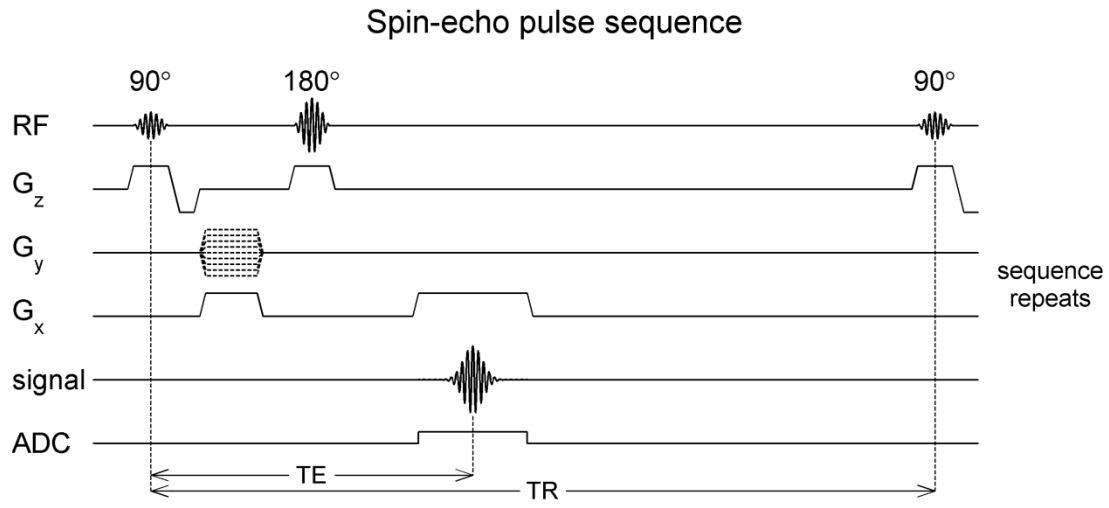
The image will be a blurred version of that obtained by fully sampling k-space. In fact, low spatial frequency components (i.e. central region of k-space) capture the overall signal intensity and shading. Higher spatial frequency components are needed to describe the fine structure and edges of an object.

12. (10pt)

a. (5pt) Draw the diagram of a spin-echo (SE) pulse sequence, including RF pulse(s), Slice selection gradients, phase-encoding gradients, frequency-encoding gradients, signal and data acquisition (ADC).

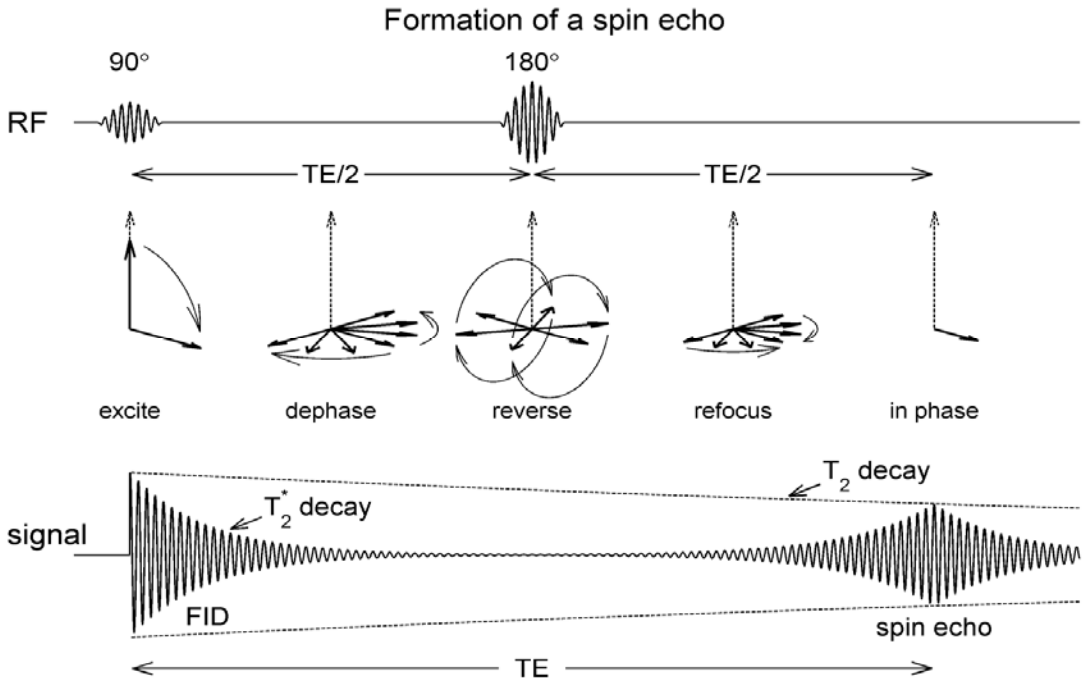
b. (5pt) Briefly explain how the spin echo (SE) pulse sequence works.

Solution:



where Gz indicates the slice selection gradients, Gy the phase encoding gradients and Gx the frequency encoding gradients.

Inhomogeneities of the static field strength (i.e. B_0), susceptibility variations, etc. cause the spins to dephase after the RF pulse. A 180-degree pulse is played after a time TE/2 to reverse the spins. As a result, the spins that rotate faster, and were ahead of the others, will end up behind the others. As they still rotate faster, they will catch up and return in phase after another interval of length TE/2. The signal is acquired at the spin echo time (TE), so that these effects (i.e. T2* effects) are removed. Note that the magnetization is still subjected to T2 decay. See also illustration below.



13. (10pt)

a. Image quality depends, among other things, on the signal-to-noise ratio (SNR). Would using larger voxels (i.e. lower spatial resolution) increase or decrease the SNR of an image? Why?

Increase the SNR. If a voxel is larger, the signal come from a larger volume of the object, which contains more spins and therefore result in higher signal.

b. A way of increasing the SNR is to acquire the same data multiple times and average the data in k-space before reconstructing the image. Assuming that the noise is white, Gaussian and statistically independent between acquisitions, show mathematically by how much the SNR increases if we use four averages of the data. What is the disadvantage of doing that?

The mean of the raw k-space signal ($S_{ave}(k)$), which is the numerator of SNR, does not change after averaging:

$$S_{\text{ave}}(k) = \frac{1}{N_{ave}} \sum_{i=1}^{N_{ave}} S_{\text{i}}(k) = \frac{1}{N_{ave}} \left(N_{ave} S(k) \right) = S(k)$$

The variance of the raw k-space signal ($\sigma^2(k)$), which is the same for each average because of the noise properties, is smaller after averaging:

$$\sigma^2_{\text{ave}}(k) = \text{var}(S_{\text{ave}}(k)) = \frac{1}{N^2_{ave}} \sum_{i=1}^{N_{ave}} \text{var}(S_{\text{i}}(k)) = \frac{\sigma^2(k)}{N_{ave}}$$

Therefore the SNR is higher, compared to the un-averaged case:

$$SNR_{\text{ave}}(k) = \frac{S_{\text{m,ave}}(k)}{\sqrt{\sigma^2_{\text{m,ave}}(k)}} = \sqrt{N_{ave}} \frac{S(k)}{\sqrt{\sigma^2_{\text{m}}(k)}} = \sqrt{N_{ave}} SNR(k)$$

If we use four averages (i.e. $N_{ave} = 4$), the SNR will be higher by a factor of two.

The disadvantage is that the acquisition is four times longer.