EL5823/BE6203 Medical Imaging I, Fall 2011, Professor Yao Wang

Midterm Exam, 10/27/2011, 2:00-4:40PM
(closed book, 1 sheet of notes double sided allowed)

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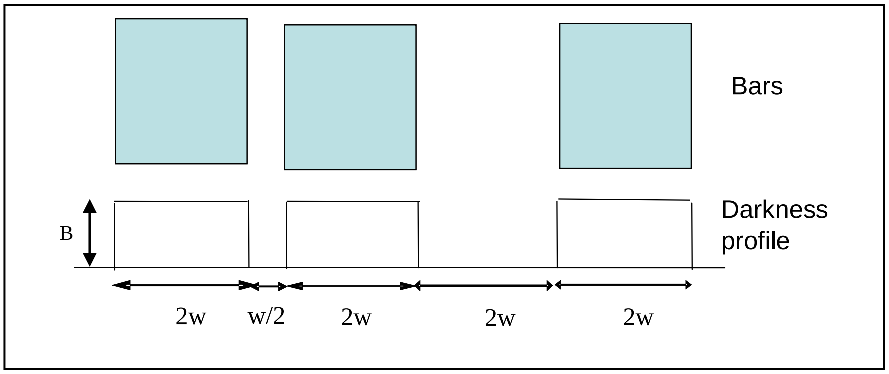
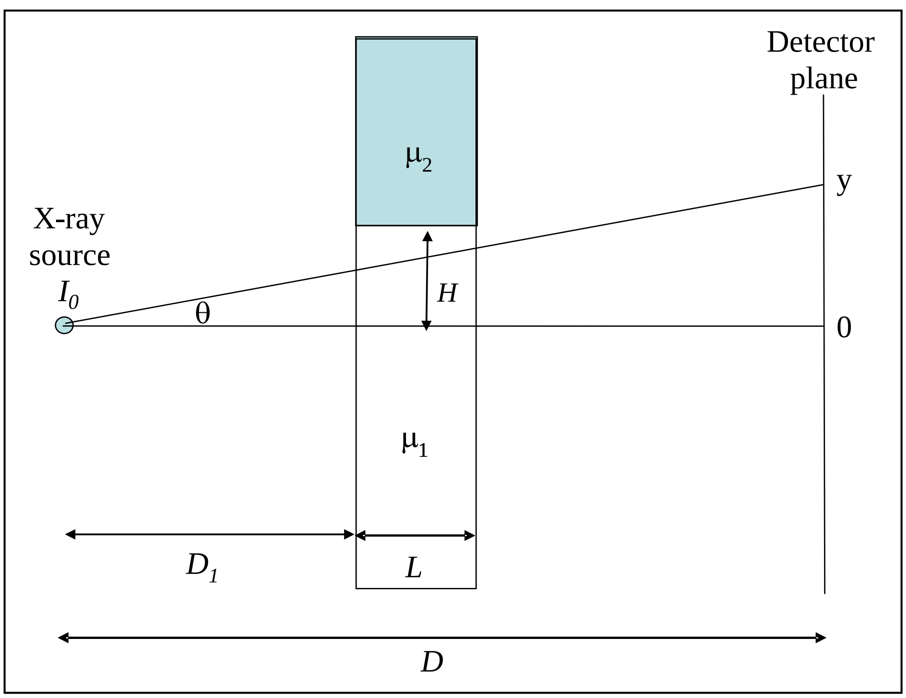
1. (10 pt) The line spread function of an imaging system is described by a rectangular box function of width W millimeter (mm), i.e., $l(x) = \begin{cases} 1/W, & |x| < W/2 \\ 0, & otherwise \end{cases}$. (a) What is the resolution of this imaging system in terms of FWHM (full width at half maximum) and in terms of lines/mm? (b) Suppose the field to be imaged contains 3 parallel bars of width $2W$ mm, spread non-uniformly, with the darkness profile as indicated in the figure below. Determine the darkness profile of the bars after imaging. Can you still tell all the bars apart? How many bars will you see? What will be their respective width?

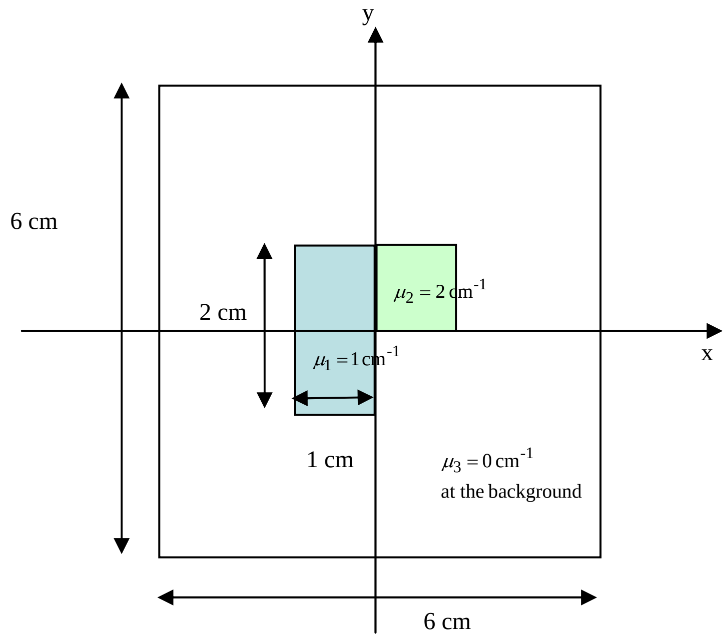


2. (20 pt) Consider the x-ray imaging of a slab that consists of two different materials with different linear attenuation coefficients μ_1 and μ_2, respectively, as illustrated below. Determine the intensity of detected photons along the y axis on the detector plane. Express your solution in terms of the θ-coordinate. You should consider the inverse square law and the oblique effect. Assume the x-ray source is an ideal point source with intensity I_0. For simplicity, assume the slab is infinitely long in the y direction.

Hint: your solution may be different for different range of θ. Specify the boundary of each range and the expression in each range clearly.



3. (25 pt) Suppose the tissue slice (with dimension 6x6cm) being imaged by a parallel beam x-ray CT scanner contains distribution of the linear attenuation coefficients as shown below. (a) Assume the detector is a point detector. Sketch the projection $g(l,\theta)$ as a function of l, for θ=0 and 90 degrees, respectively. You should indicate the magnitudes of the projected values where necessary on your sketch. Also clearly specify any transition points in the l-axis. (b) Sketch the image obtained by backprojections from 0 and 90 degrees, respectively, and by sum of the two backprojections. You should assume that you know the dimension of the tissue being imaged and normalize your backprojection using the known dimensions as shown in the figure. (c) Determine the 2D Fourier transform of the original image along a horizontal line passing the origin in the frequency plane, through the 1D Fourier transform of the projection signal. (d) What will be the projected function for θ=0 if the detector is an area detector with width 0.2 cm. Sketch the projection function. (e) How is the image recovered from the readings obtained with such a detector related to the image recovered from using an ideal point detector? (assume that in both cases you use the convolution projection method with the same filter). Note: if you need to know the Hankel transform of a function to describe your filer, you can just indicate the filter is the Hankel transform of that function.



4. (10 pt) (a) In nuclear imaging, what type of radiotracer is needed for SPECT? (b) What type is needed for PET? (c) Do we need to use a collimator in SPECT camera? What about for PET? Why? (d) What is the desirable range of the half-hour life of the radiotracer for medical imaging? Why?

5. (10pt) Recall that the detector reading in SPECT can be described by

$$\phi(\ell,\theta) = \int_{-\infty}^{R} \frac{A(x(s),y(s))}{4\pi(s-R)^2} \exp\left\{-\int_{s}^{R} \mu(x(s'),y(s');E)ds'\right\} ds$$

Suppose you already know the distribution of the linear attenuation coefficients μ(x,y) of the body slice being imaged through prior CT imaging. Describe the steps you need to recover the radio activity distribution A(x,y) from the detector readings at all (l,θ) pairs.

6. (25 pt) A 2-D slice to be imaged is shown below, which consists of three regions R1, R2, R3. The linear attenuation coefficients in the regions are $\mu_1 = 1cm^{-1}$, $\mu_2 = 2cm^{-1}$, $\mu_3 = 3cm^{-1}$

a. Suppose a solution containing a gamma ray emitting radio tracer with an initial radioactivity of A_0 fills section R1. We image the radioactivity distribution in this slice using a rotating SPECT camera at time *t* after the injection of the radio tracer. Compute the measured signal by the camera at positions A and B, respectively. Assume the signal is measured at time *t* after the injection of the radionuclide solution. Assuming the half life for this radionuclide is *T*.

b. Now suppose the radio tracer in (a) is replaced by a positron emitting radio tracer with the same radio activity, with the same initial radioactivity and half-life. This time the slice is imaged using a PET scanner. Compute the measured signal by the pair of cameras positioned at A and B.