EL5823/BE6203 Fall09 final Exam Solution.

1. Solution: a) we define: $B(l.\theta.S) = \frac{1}{4\pi(S-R)^2} \exp\{-\int_S^R \mu(x(S'),y(S'),E)\,dS'\}$

for each $l.\theta.S$. we can calculate $B(l.\theta,S)$. since we already know $\mu(x.y)$.

Let $A'(x(s).y(s);l.\theta,S) = \frac{A(x(s).y(s))}{B(l.\theta,S)}$. then

$$\phi(l.\theta) = \int_R^R A'(x(S).y(S);l,\theta,S)\,dS.$$

By implement filter or convolution Backprojection to $\phi(l.\theta)$ we can get $A''(x.y)$. since $A''(x.y) \neq A'(x(S).y(S);l.\theta,S)$ we just assume $A'(x(S)\,y(S),l.\theta,S) \simeq A''(x.y)$ for every $l.\theta.S$. and just average them to get $A(x.y)$

$$A(x.y) = \cancel{mean}\ A''(x.y)\times mean\left(\frac{1}{B(l.\theta,S)}\right)\Bigg|_{\substack{x=l\cos\theta - s\sin\theta \\ y = l\sin\theta + s\cos\theta}}$$

b) one can use an iterative scheme.
starting with the $\mu(x.y)$ obtained with x-ray CT.

1). obtain first estimation of $A(x.y)$ using the method of part a)

2) For given $l.\theta,S$. Set
$g(l.\theta;-R) = \exp\{\int_{-R}^R \mu(x(S')\,y(S'))dS'\} = \frac{A'(x(-R)y(-R))}{A(x(-R)y(-R))} \cdot 4\pi 4R^2$

recover $\mu(x\,y)$ from $g(l.\theta.-R)$ for all $l.\theta$ ($\mu'(x.y)$) using filtered back projection.

3). go to step 1. by using the new estimation of $\mu(x.y)$ till
$|\mu'(x.y) - \mu(x.y)| < \varepsilon$.

2. Solution. since $T_{1/2} = 8$ hour

$$\lambda = \frac{0.693}{T_{1/2}} \Rightarrow A = A_0 e^{-\lambda t} = 1 \times e^{-\frac{0.693}{8} \times 2} \ mCi/cm^2 = \frac{1}{\sqrt[4]{2}} \ mCi/cm^2$$

A)

$$\phi_A = \int_{-2}^{0} \frac{A}{4\pi(3-x)^2} e^{-\mu_2(-x) - \mu_3 \cdot 2 - \mu_1 \cdot 1} dx + \int_{1}^{2} \frac{A}{4\pi(3-x)^2} e^{-\mu_1(2-x) - \mu_3 \cdot 1} dx$$

$$\phi_B = \int_{-2}^{0} \frac{A}{4\pi(x+3)^2} e^{-\mu_2(x+2) - \mu_3} dx + \int_{1}^{2} \frac{A}{4\pi(x+3)^2} e^{-\mu_2 \cdot 2 - \mu_1(x-1) - \mu_3 \cdot 2} dx$$

$$\phi_C = \int_{-1}^{1} \frac{A}{4\pi(3-y)^2} e^{-\mu_2(1-y) - \mu_3 \cdot 2} dy$$

since the object is symmetric in y-direction. So. $\phi_D = \phi_C$

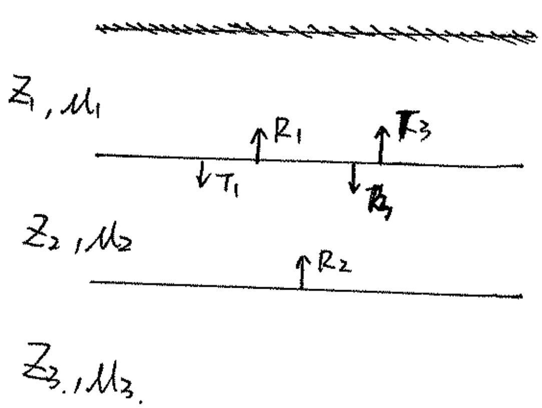
B)

$$\phi_{AB} = k \cdot \left(\int_{-2}^{0} A dx + \int_{1}^{2} A dx \right) \cdot e^{-[\mu_1 + 2\mu_2 + 3\mu_3]}$$

$$= 3kA \, e^{-[\mu_1 + 2\mu_2 + 3\mu_3]}$$

$$\phi_{CD} = k \cdot \int_{-1}^{1} A dx \cdot e^{-\mu_2 \cdot 2} e^{-\mu_3 \cdot 4} = 2kA \, e^{-[4\mu_3 + 2\mu_2]}$$

3. Solution:

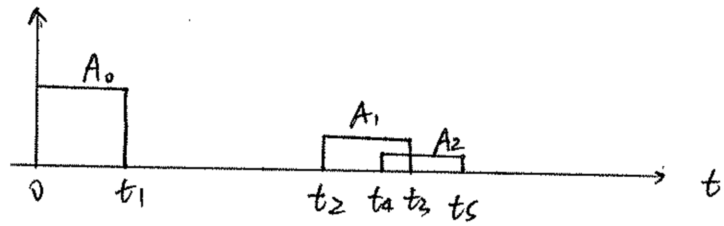


$$T_1 = \frac{2Z_2}{Z_1 + Z_2};$$

$$R_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1};$$

$$R_2 = \frac{Z_3 - Z_2}{Z_3 + Z_2};$$

$$T_3 = \frac{2Z_1}{Z_1 + Z_2};$$



$A_0 = 1$

$t_1 = 10\mu s$

$A_1 = A_0 \cdot R_1 \cdot e^{-\mu_1 \cdot 2d_1} = \frac{1}{3} e^{-0.12} = 0.296$

$t_2 = \frac{2d_1}{C_1} = 40\mu s$

$t_3 = t_2 + 10\mu s = 50\mu s$

$A_2 = A_0 \cdot T_1 \cdot R_2 \cdot T_3\, e^{-(\mu_1 \cdot 2d_1 + \mu_2 \cdot 2d_2)}$

$= \frac{8}{45} e^{-0.15} = 0.153.$

$t_4 = t_2 + \frac{2d_2}{C_2} = 46.5\mu s$

$t_5 = t_4 + 10\mu s = 56.5\mu s$

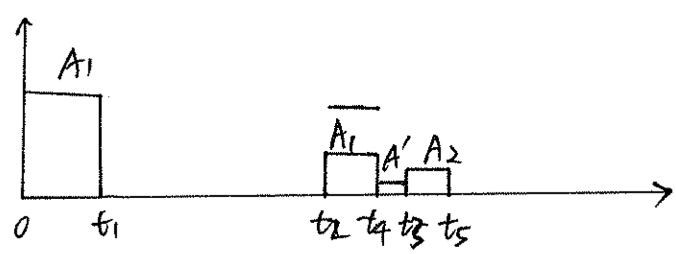
So the full expression for the received signal will be

$$r(t) = A_0 \mathrm{rect}\left(\frac{t - t_1/2}{10}\right) + A_1 \mathrm{rect}\left(\frac{t - t_1/2 - t_2}{10}\right) \cos 2\pi f\left(t - \frac{t_1}{2} - t_2\right)$$
$$+ A_2 \mathrm{rect}\left(\frac{t_1 - t_1/2 - t_4}{10}\right) \cos 2\pi f\left(t - \frac{t_1}{2} - t_4\right)$$

for the overlapping part. the envelop of the signal will be

$$A' = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(2\pi f \cdot 2d_2/C_2)}$$
$$= A_1 - A_2 = 0.143$$

← because the angle $2\pi f \frac{d_2}{C} 2$ is multiple of π

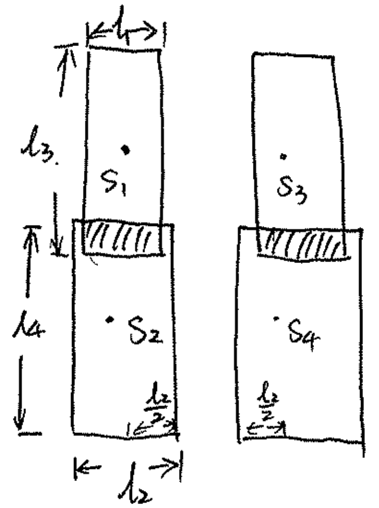


4. Solution: $\lambda = \frac{c}{f} = \frac{1540 m/s}{3MHz} = 5.13 \times 10^{-4} m.$

$D^2/\lambda = (1cm)^2 / 5.13\times10^{-4} m = 19.5 cm > d_1$

$\Rightarrow S_1\ S_2$ in the Fresnel Region.

$19.5 < d_1 + d_2 \Rightarrow S_2 . S_4$ in the Fraunhofer Region.



$l_1 = 1cm$

$l_2 = \frac{\lambda z}{D} = \frac{5.13\times10^{-4} m \times 20\times10^{-2} m}{1\times10^{-2} m} = 1.026 cm$

$l_3 = l_4 = \frac{Tc}{2} = 0.154 m = 15.4 cm$

b) S_1, S_2. can't be seperated. they are overlapped.

c). S_1, S_3 can be seperated. since the interval between them is $d_3 - l_1 = 0.5cm$

d) $S_2 . S_4$ can be seperated. Since the interval between them is $d_3 - l_2 = 0.474 cm$

5. Solution: a) $t_i = iT = i d \sin\theta / c$

b) if T_0 fires at time 0.

T_2 should fire at $t_2 = 2T = 2d\sin\theta / c$

b) Solution: a) $\Delta \nu = \Delta z \cdot \gamma G_z = 4.258\ kHz/gauss \times 1\ mm \times 3\ gauss/mm$

$= 12.8\ kHz$

$\bar{\nu} = \gamma (B_0 + G_z \cdot z)$

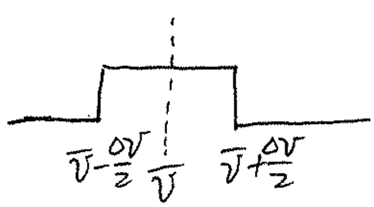
$= 4.258\ kHz/gauss \times (1.5 \times 10^4\ gauss + 3\ gauss/mm \times 10\ mm)$

$= 6.64 \times 10^4\ kHz$.

So the frequency range of the RF waveform should be

$\left(\bar{\nu} - \frac{\Delta\nu}{2},\ \bar{\nu} + \frac{\Delta\nu}{2} \right)$

the desired "ideal" spectrum of the RF waveform



b) For the "ideal" waveform:

$s(t) = A \cdot \Delta\nu\ \mathrm{sinc}(\Delta\nu t) e^{j2\pi\bar{\nu}t}$.

c). For the non-ideal waveform.

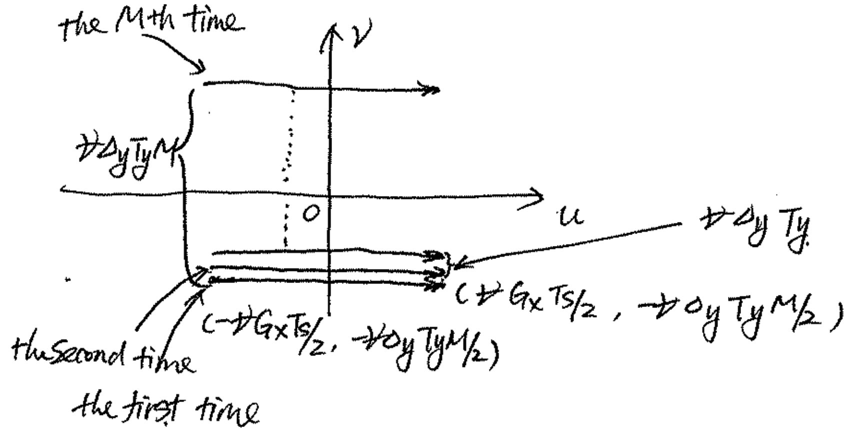
$s'(t) = s(t) \cdot \mathrm{rect}(t/T)$

In the frequency domain.

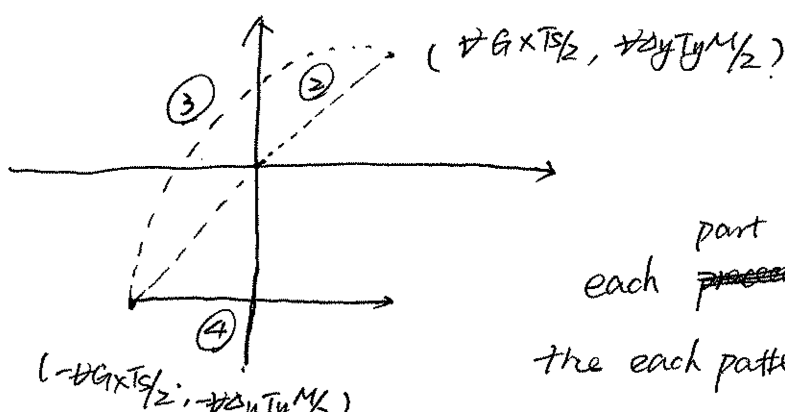
$S'(\nu) = S(\nu) * \frac{1}{T}\mathrm{sinc}(\nu)$

d). Beyond the intended slice, other areas are also excited. since the spectrum of these area is not zero. So the collected signal will be affected by these area.

7). Solution: a) In the frequency domain. the sampling pattern works as below.



Next. using the first read-time to show how it works.



each ~~proceed~~ part corresponding to the each pattern of the pulse sequency shown in the picture.

b) Since. we use spin-echo here to measure the signal. So the measured signal reflects T_2 decay.

c) since

$$\begin{cases} FOV_x = W = \frac{1}{\Delta u} = \frac{N}{\gamma G_x T_s} & ① \\ FOV_y = W = \frac{1}{\gamma \Delta_y T_y} & ② \\ \Delta x = \frac{1}{\gamma G_x T_s} = \Delta & ③ \\ \Delta y = \frac{1}{\gamma \Delta_y T_y M} = \Delta & ④ \end{cases}$$

$$\text{So. } \frac{N}{\cancel{\gamma} G_x T_s} = \frac{1}{\cancel{\gamma} \Delta_y T_y} = W \Rightarrow \Delta_y T_y = \frac{T_s}{N} G_x = \frac{1}{\gamma W} \quad ⑤$$

$$\text{from ③} \quad \Delta x = \Delta = \frac{1}{\gamma G_x T_s} = \frac{W}{N}$$

$$N = \frac{W}{\Delta}$$

$$\text{from ④} \quad \Delta y = \Delta = \frac{W}{M} \qquad M = \frac{W}{\Delta}$$

$$\text{so } \Delta_y T_y = \frac{1}{\gamma W}$$

$$T_s G_x = \frac{N}{\gamma W} = \frac{1}{\gamma \Delta}$$

So the equations that the parameters G_x. ΔG_y. must satisfy.

$$\begin{cases} M = N = \frac{W}{\Delta} \\ \Delta G_y T_y = \frac{1}{\gamma W} \\ T_s G_x = \frac{1}{\gamma \Delta} \end{cases}$$

d) the demension of the reconstructed image is $N \times M$. (and $M = N = \frac{W}{\Delta}$)