EL582/BE620 Medical Imaging I, Fall 2009, Professor Yao Wang
(closed book, 2 sheets of notes double sided allowed)
12/17/2009, 1:00-3:50PM

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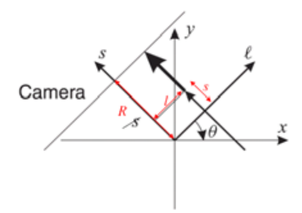
1. (10 pt) (a) In SPECT, the measured signal is related to the radioactivity distribution $A(x, y)$ and attenuation distribution $\mu(x, y)$ as follows, where (x, y) are related to (l, θ) as indicated in the figure below. (a) Suppose you already know $\mu(x, y)$. Describe one algorithm that you can use to you recover $A(x, y)$ from the measured data captured at many values of (l, θ). (b) Suppose you don't know $\mu(x, y)$ exactly, but have some initial guess by using X-ray CT. How would you go about recovering $A(x, y)$ and $\mu(x, y)$?

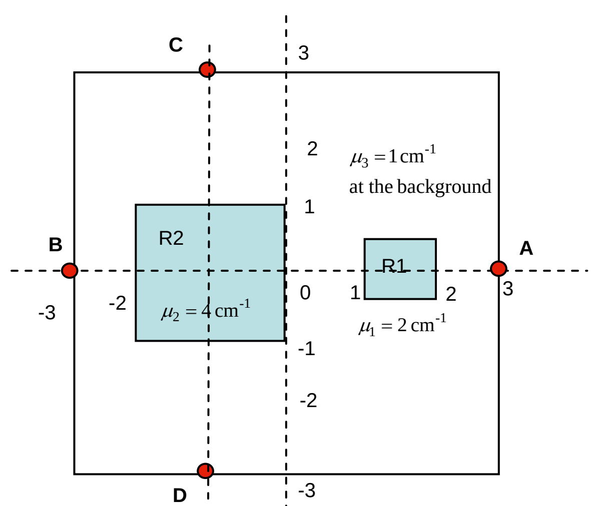


$$\phi(\ell, \theta) = \int_{-\infty}^{R} \frac{A(x(s), y(s))}{4\pi(s-R)^2} \exp\left\{-\int_s^R \mu(x(s'), y(s'); E)\,ds'\right\} ds$$

$$x(s) = \ell\cos\theta - s\sin\theta$$
$$y(s) = \ell\sin\theta + s\cos\theta$$

2. (20 pt) A 2-D slice to be imaged is shown below.

   a. Suppose a solution containing a gamma ray emitting radionuclide (with a half life of 8 hour) with concentration of A=1 mCi/cm^2 fills section R1 and R2. We image the radioactivity distribution in this slice 2 hour after the injection of the radionuclide solution, using a rotating SPECT camera. Compute the measured signal by the camera at positions A, B, C, D, respectively.
   b. Now suppose the radionuclide in (a) is replaced by a positron emitting radionuclide with the same concentration. This time the slice is imaged using a PET scanner. Compute the measured signal by the cameras positioned at A and B, and the signal by the cameras positioned at C and D.



3. (15 pt) Consider an ultrasound imaging scenario illustrated in Fig. P3. Assume the media has three layers, with their depths indicated in the figure, with d1=3cm, d2=0.5cm. The acoustic impedance, speed of sound, and attenuation coefficients of different media are denoted by $Z_i; c_i; \mu_i, i = 1,2,3$, respectively. The values are given in Table P3. Assume the transducer generates an acoustic wave with envelop being a rectangular pulse of duration T=10 μs at frequency of 3 MHz and an amplitude of 1. Assume that there is no reflection between the transducer and the medium it resides in (medium 1). (a) write down an expression for the signal received by the transducer. (b) sketch the A-mode signal (envelope of the received signal) received by the transducer. Note that you only need to consider waves that enter a surface in the normal directions. You should take into account of signal attenuation in distance.

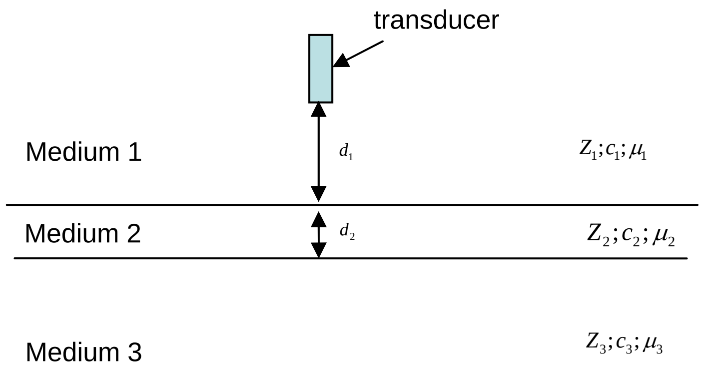
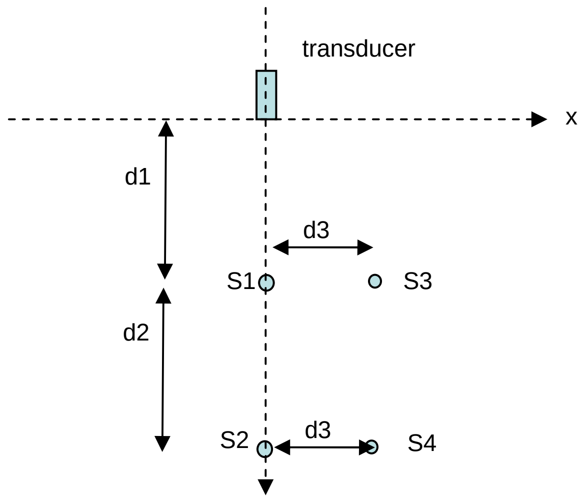


Figure P3

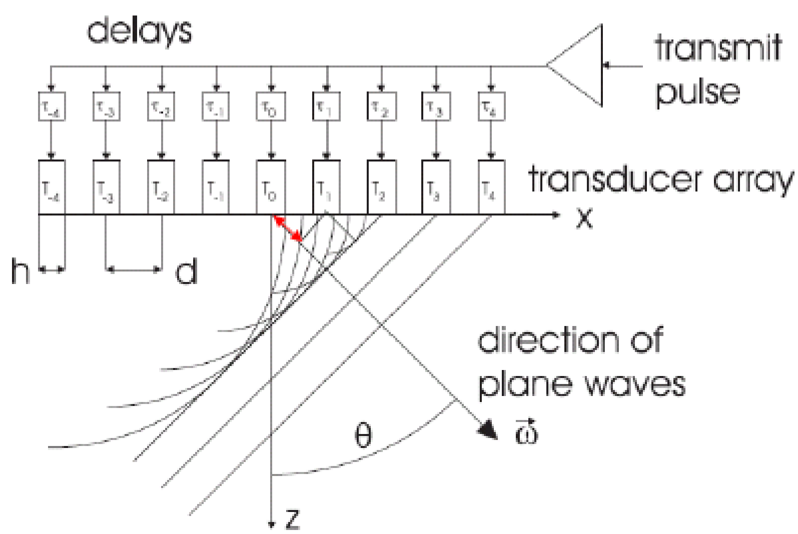
	Acoustic Impedance Z (kg/m^2 sec)	Speed of sound c (m/sec)	Attenuation factor (cm^-1)
Medium 1	1 E6	1500	0.02
Medium 2	2 E6	1550	0.03
Medium 3	3 E6	2000	0.04

Table P3

4. (15 pt) A flat face transducer with a circular face with diameter D (D=1cm) is used to image a medium with four point scatters with positions indicated in the figure below. The distances d1=10cm, d2=10 cm, d3=1.5cm. The medium has a speed of sound c=1540 m/s. The resonance frequency of the transducer is 3 MHz. The narrowband signal generated by the transducer has a rectangular envelope of width T=200 μs.
   a. Sketch roughly the measured reflectivity distribution (i.e. the obtained ultrasound image) in the x-z plane by the transducer working in the B-mode. Indicate the dimension of any object that may be seen by the scanner.
   b. Can we separate the two scatters S1 and S2 based on the image? Why?
   c. Can we separate the two scatters S1 and S3? Why?
   d. Can we separate the two scatters S2 and S4? Why?



5. (5 pt) Consider the following transducer array. If we would like the beam to propagate in a direction with angle q, what should be the delay between the firing of the transmit pulses between the transducers? Specifically, if T_0 fires at time 0, when should T_2 fires?



6. (15 pt) Consider a MRI session. Assume the static field strength is B_0=1.5Tesla, the z-gradient is Gz=3 gauss/mm, and the gyromagnetic ratio is γ =4.258 KHz/gauss. We would like to image a slice with center position $\bar{z}$ =20cm, thickness Δz =1mm. (a) Find the frequency range of the RF waveform required to excite this slice and sketch the desired "ideal" spectrum of the RF waveform. (b) Write down the mathematical expression for the "ideal" waveform that will yield the desired spectrum. (c) In practice, we truncate this "ideal" waveform with some finite-duration window function. Suppose we use a rectangular window of duration T, sketch the spectrum of this non-ideal RF waveform. (d) What is the impact of using this non-ideal excitation waveform in terms of slice selection?

7. (20 pt) Consider an MRI scan using a pulse sequence shown below. You use a constant x-gradient G_x during each readout time, and take N samples during each read-out gradient of duration T_s. Also you start with the y-gradient at $G_y = \Delta_y M/2$, and decrement the y-gradient by Δ_y each time, and you repeat the process M times. The scan parameters are chosen so that $\gamma G_x T_s = \gamma \Delta_y T_y M$. (a)What are the sampling patterns in the Frequency domain? (b) Would the measured signal reflect T2 decay or T2* decay? (c) If you would like to have a pixel resolution of ($\Delta x, \Delta y$) with $\Delta x = \Delta y = \Delta$ and covers an area of $FOV_x \times FOV_y$, with $FOV_x = FOV_y = W$, what are the constraints on the scan parameters? (i.e. give the equations that the parameters $G_x, \Delta G_y, T_s, T_y, M, N$ must satisfy in terms of Δ, W). (d) What is the reconstructed image dimension?