EL5823/BE6203 Medical Imaging I, Fall 2007, Professor Yao Wang

Midterm Exam, 10/16/2008, 3:00-5:50PM
(closed book, 1 sheets of notes double sided allowed)
No peeking into your neighbors. Cheating will result in a failing grade for the course.

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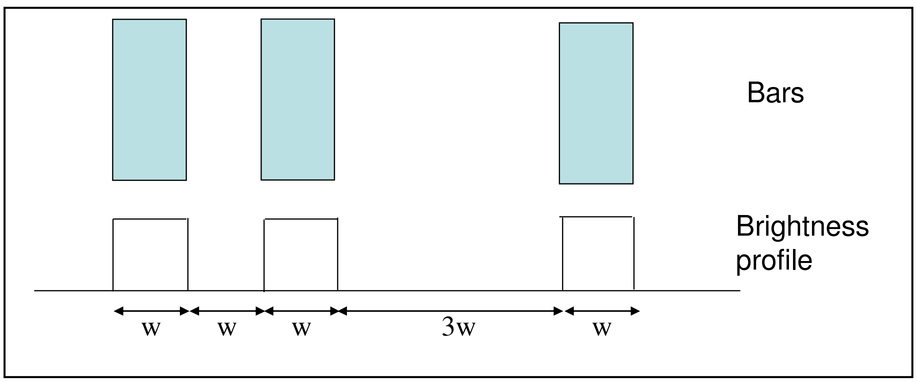
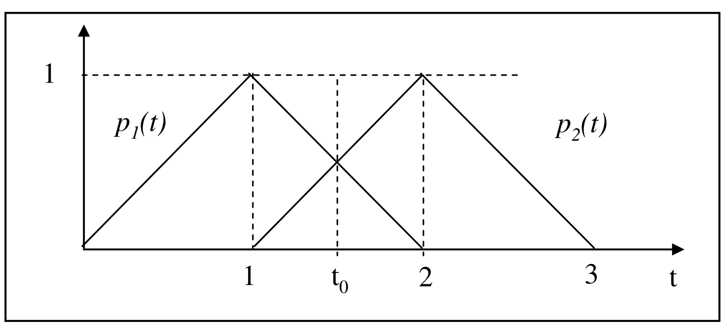
1. (15 pt) The line spread function of an imaging system is described by a rectangular box function of width W millimeter (mm), i.e., $l(x)=\begin{cases}1, & |x|<W/2\\ 0, & otherwise\end{cases}$. (a) What is the resolution of this imaging system in terms of FWHM (full width at half maximum) and in terms of lines/mm. (b) Suppose the field to be imaged contains 3 parallel bars of width W mm, spread non-uniformly, with the darkness profile as indicated in the figure below. Determine the darkness profile of the bars after imaging. Can you still tell all the bars apart? How bars will you see? What will be their respective width?

   Hint: the convolution of two rectangular functions of the same width becomes a triangle function.

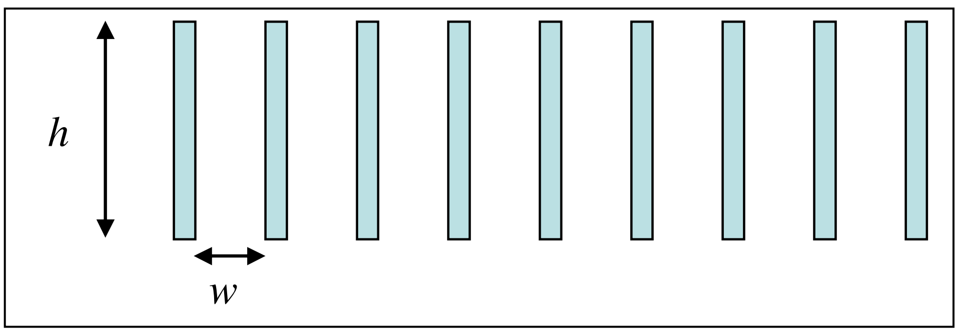


2. (15 pt) Suppose a disease is diagnosed based on a single blood test and that the distribution of the test result t among the normal population is a triangular function $p_1(t)$ and that for people with the disease is another triangular function $p_2(t)$, both illustrated in the figure below. Suppose the patient is considered to be normal if the test result is less than t_0, and abnormal otherwise, with t_0, as indicated in the figure. Determine the sensitivity (true positive fraction), specificity (true negative fraction) and diagnostic accuracy.



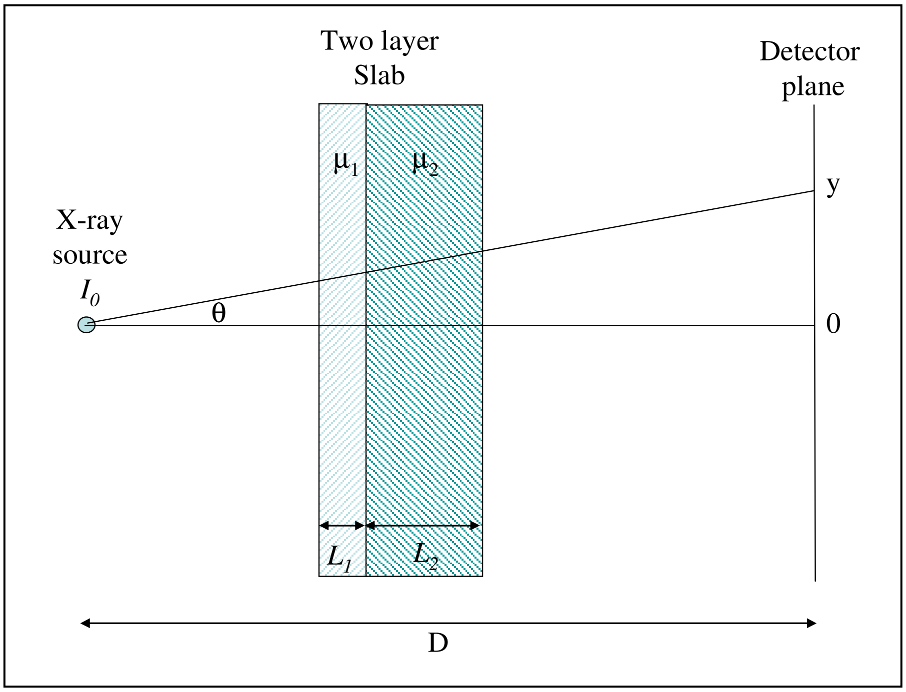
3. (15 pt) (a) Describe briefly how X-ray is created in an X-ray tube, including the two primary form of radiation. (b) If the voltage applied between the anode and the cathode is 150 KV, and the binding energy of K-shell electrons in the tungsten anode is 70 KeV, and that of L-shell and M-shell are respectively 11 KeV and 3 KeV. Sketch the shape of the resulting x-ray energy spectrum. Properly label your horizontal axis. You only need to consider characteristic rays generated when ejected K-shell electrons are replaced by L-shell and an M-shell electrons. (c) Sketch the energy spectrum created when the voltage is changed to 50 KV. Your curves for (b) and (c) should have proper scales relative to each other. Would there be characteristic rays in case (c)? Why?

4. (15 pt) (a) In X-ray imaging, why are photons undergoing Compton scattering undesired? (b) One way to avoid counting scattered photons is by placing an anti-scatter grid (with spacing *w* and height *h*) on top of the X-ray detector, shown below. Determine the maximum scattering angle of X-rays that can pass through the grid and hence being detected. (c) Suppose the maximum and minimum of the detected signals without scattering are respectively f_{max} and f_{min}. With scattering, both increase by f_s. Describe how does f_s affect the global contrast of the resulting image.



5. (10 pt) Backprojection summation is a method to reconstruct an image from its parallel projections. Can you get an accurate reconstruction using this method (assuming the spacing between projection angles is sufficiently small and that the spacing between adjacent lines under the same projection angle is also sufficiently small). If not, what is difference between the reconstructed image by this method and that by filtered backprojection using the ideal filter? Can you characterize this difference in both the frequency domain and in the image domain?

6. (10 pt) Consider the x-ray imaging of a two-layer slab, illustrated below. Determine the intensity of detected photons along the *y* axis on the detector plane. Express your solution in terms of the y-coordinate Sketch this function. You should consider the inverse square law and the oblique effect. Assume the x-ray source is an ideal point source with intensity I_0. For simplicity, assume the slab is infinitely long in the *y* direction.



7. (20 pt) The tissue slice being imaged by a parallel beam x-ray CT scanner is shown in the figure below. (a) Assume the detector is a point detector. Sketch the projection $g(l,\theta)$ as a function of l, for θ=0, 45, 90, and 135 degrees, respectively. You should indicate the magnitudes of the projected values where necessary on your sketch. Also any transition points in the horizontal axis. (b) Sketch the image obtained by backprojections from both 0 and 90 degree projections. You should normalize your backprojection using the dimension of the imaged region as indicated on the figure. (c) What will be the projected function for θ=0 if the detector is an area detector with width 0.1 cm. Sketch the projected function. (d) Determine the Fourier transform of the original image along a line with orientation θ=90 degree.

Hint: (1) The Fourier transform of the rectangular function is given by:

$$g(x) = \begin{cases} 1 & |x| < W/2 \\ 0 & otherwise \end{cases} \Leftrightarrow G(f) = \frac{\sin \pi W f}{\pi f}$$

(2) Shifting property of Fourier transform: $g(x - x_0) \Leftrightarrow G(f)e^{j2\pi x_0 f}$