

Midterm Exam, 10/30/06, 3:40-5:50PM  
(closed book, 2 sheets of notes allowed)

1. (15 pt) (a) What determines the highest energy of x-ray photons emitted from an x-ray tube? Sketch the typical shape of the x-ray energy spectrum. (b) In x-ray imaging for medical applications, why are low-energy photons undesired? What measures can be taken to reduce the number of the low-energy photons entering the human body? (c) why are photons undergoing scattering undesired? What measures can be taken to prevent these photons from entering the detectors?
2. (15 pt) A slab of soft tissue with one blood vessel in the middle is imaged under an x-ray imaging system, as shown in Figure P.2. The blood vessel may be injected a contrast agent. For ease of computation, assume the tissue and the vessel both have square shaped cross-sections, with dimensions shown in the figure. Further, we assume photons are uniformly shed upon the side of the tissue. For the range of the photon energy of the x-ray used, the linear attenuation coefficients of the soft tissue, the blood vessel and the contrast agent are  $0.4 \text{ cm}^{-1}$  and  $0.2 \text{ cm}^{-1}$ , and  $20 \text{ cm}^{-1}$ , respectively. Determine the local contrast of the blood vessel (a) when the contrast agent is not injected and (b) when the contrast agent is injected.

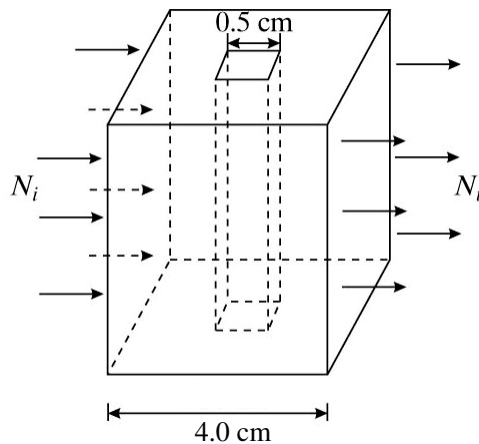


Figure P.2

3. (15 pt) Consider the x-ray imaging of a cube, illustrated in Figure P.3. Determine the intensity of detected photons along the y axis on the detector plane. Express your solution in terms of the angle  $\theta$ . Sketch this function. You should consider the inverse square law and the oblique effect. Assume the x-ray source is an ideal point source with intensity  $I_0$ , and the object has a constant linear attenuation coefficient  $\mu$ .

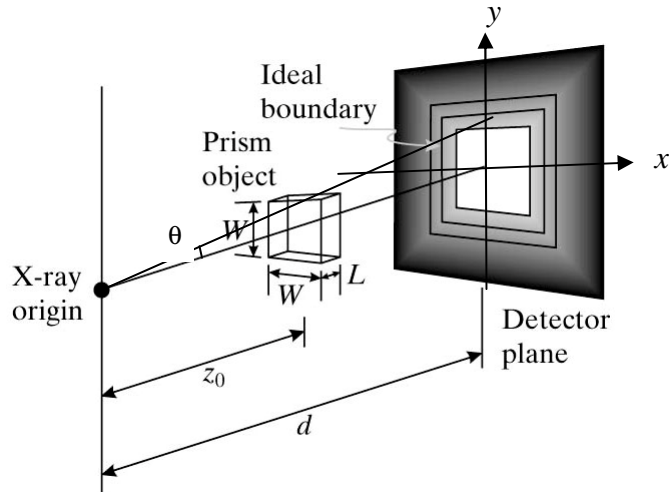


Figure P.3

4. (10 pt) Describe the major steps in the following three reconstruction algorithms from parallel projections: backprojection, filtered backprojection, and convolution backprojection. Compare these methods in terms of accuracy and complexity.
5. (30 pt) Suppose the tissue slice being imaged by a parallel beam x-ray CT scanner contains a rectangular target (see Figure P.5) with a constant attenuation coefficient  $\mu=1 \text{ cm}^{-1}$ . (a) Assume the detector is a point detector. Sketch the projection  $g(l, \theta)$  as a function of  $l$ , for  $\theta=0, \arctan(1/2), 45, 90$  degrees, respectively. You should indicate the magnitudes of the projected values and the  $l$ -direction coordinate for all transition points on your sketch. (b) Sketch the image obtained by backprojection from 0 and 90 degree projections. You should assume that the dimension of the tissue being imaged is  $4 \times 4 \text{ cm}$  and normalize your backprojection using the known dimensions. (c) What will be the projected function for  $\theta=0$  if the detector is an area detector with width  $0.1 \text{ cm}$ . Sketch the projection function. (d) Determine the Fourier transform of the original image along a line with orientation  $\theta=0$  degree; (e) repeat (d) for  $\theta=45$  degree.

Hint: The Fourier transform of the rectangular function is given by:

$$g(x) = \begin{cases} 1 & |x| < W/2 \\ 0 & \text{otherwise} \end{cases} \Leftrightarrow G(f) = \frac{\sin \pi W f}{\pi f}$$

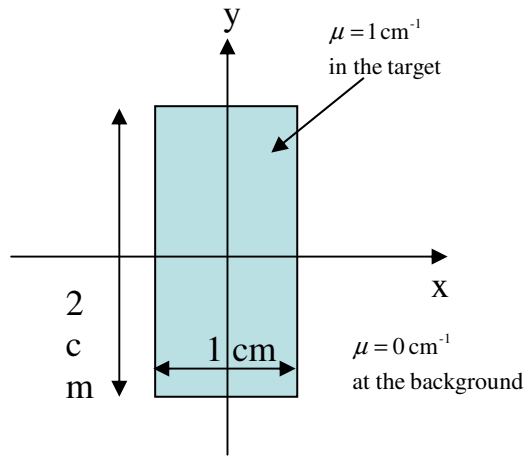


Figure P.5

6. (5 pt) The line spread function of an imaging system is described by

$l(x) = \frac{1}{4} \exp\left\{-\frac{x^2}{4}\right\}$ . Determine its FWHM (full width at half maximum) and resolution (lines/cm) (assuming  $x$  has a unit of cm).

7. (10 pt) Assuming the probability density functions of a test result for patient with and without a disease are described by exponential distributions:

$$\text{Normal} : p_x(x) = \frac{1}{2} \exp\{-|x - \mu_n|\}; \quad \text{Diseased} : p_x(x) = \frac{1}{4} \exp\{-2|x - \mu_d|\};$$

Assume the diagnosis is determined based on a threshold  $t$ . For a patient with test value below  $t$ , we call it normal. Otherwise, we call it diseased. Determine the sensitivity (true positive fraction), specificity (true negative fraction) and diagnostic accuracy in terms of  $t$ . Assume  $\mu_n = 5, \mu_d = 10, \mu_n < t < \mu_d$ .

Note: if you don't have enough time to complete the calculation, please make sure that you at least write down the integral equations clearly.