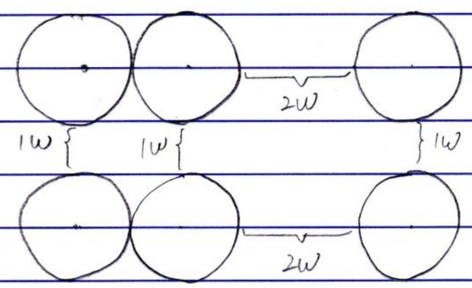
EL5823/BE6203/G16.4426 Medical Imaging, Spring 2014, Midterm Solution

1.

(a) both the minimal horizontal and vertical distance is w

(b)

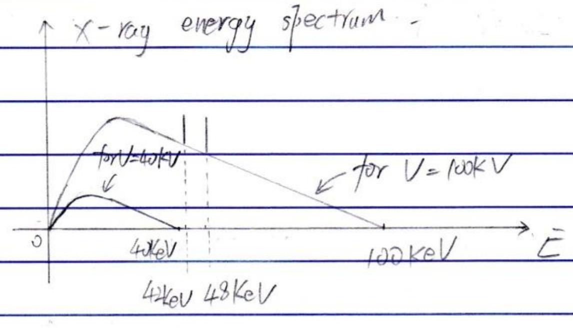


Yes, this system can tell all objects apart.

2.

(a) $\Delta E_1 = 50 - 8 = 42$ KeV

$\Delta E_2 = 50 - 2 = 48$ KeV



(b)

There would be no characteristic rays, because the incident electron energy is not enough.

(c) To provide attenuation contrast otherwise may not enough to do a good medical imaging.

For example vessel ~~muscle~~ and soft tissue may have similar attenuation coefficient, say 0.3 and 0.4. It does not provide enough intrinsic contrast. By using contrast agents, the attenuation coefficient may become 5 ~~0.3~~ and 0.4 ~~5~~. It will allow us to make a good imaging.

physical reason? K-edge effect?

-1

The contrast agents have K-shell binding energy just below the x-ray photon, and hence very high attenuation coefficient in x-ray energy slightly higher than the binding energy.

3.

(a) outside body

photon

the attenuation coefficient distribution

(b) inside body

photon

radioactive distribution

4.

(a) ~~radiotracer that along with Positi~~

radiotracer with Gamma Decay. Other aspects like energy, half-life, half-value-layer that are common to radiotracer should also be considered.

(b) radiotracer with Positron Decay.

(c) we need a collimator in SPECT to deal with scattering

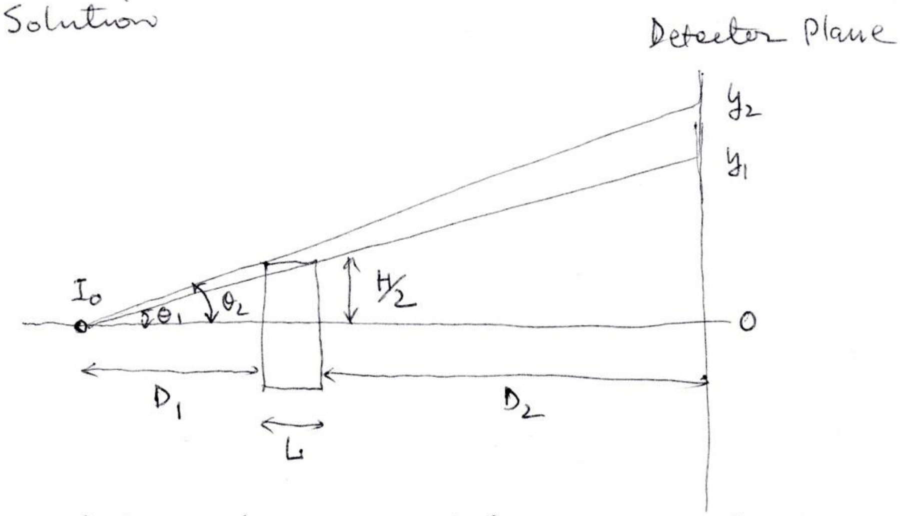
we do not ~~n~~ need a collimator in PET, because it uses Annihilation Coincidence Detection to deal with scattering and other noise.

(d) first we should choose radiotracer with proper half-life from minutes to hours. It's long enough for us to do medical imaging and not too long to keep patients to stay on the imaging equipments too long or causing other problems in their life.

If half-life is relatively short, imaging must be taken short after the ingest or injection of the radiotracer. If half-life is relatively long, patients have to ~~waiting~~ waite some time before imaging.

Another affect would be the product place of radiotracer. If half-life is short, it may need to be made just in hospital. otherwise, it may be made in other factories.

Prob 5 Solution



(a)

The detected signal follows different trends in 3 Different segments. Only positive axis is shown above

Region I: $-y_1 < y < y_1$: X-ray go through entire object with length $L/\cos\theta$

II: $y_1 < y < y_2$ & $-y_2 < y < -y_1$: X-ray go through part of obj

III: $y > y_2$ x $y < -y_2$: X-ray does not hit obj.

To determine y_1, y_2:

$$\tan\theta_1 = \frac{H/2}{D_1+L} = \frac{y_1}{D_1+D_2+L} \longrightarrow y_1 = \frac{H(D_1+D_2+L)}{2(D_1+L)}$$

$$\tan\theta_2 = \frac{H/2}{D_1} = \frac{y_2}{D_1+D_2+L} \longrightarrow y_2 = \frac{H(D_1+D_2+L)}{2(D_1)}$$

Also $\tan\theta(y) = \dfrac{|y|}{D_1+D_2+L}$, $\cos\theta(y) = \dfrac{D_1+D_2+L}{\sqrt{(D_1+D_2+L)^2+y^2}}$

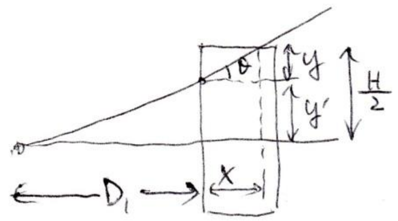
$$\sin\theta(y) = \frac{|y|}{\sqrt{(D_1+D_2+L)^2+y^2}}$$

Region I:

$$I(y) = \overset{I_s}{\frac{I_0}{4\pi (D_1+D_2+L)^2}} \cos^3\theta(y)\, e^{-\mu L/\cos\theta(y)}$$

Region II:

The x-ray intersect the object w/ length $\frac{X}{\cos\theta}$
we need to determine X:



$$\frac{H}{2(D_1+X)} = \frac{y}{D_1+D_2+L} = \tan\theta$$

$$D_1 + X = \cancel{\frac{H}{2(D_1+D_2+L)}} \quad \frac{H}{2}\frac{(D_1+D_2+L)}{y}$$

$$X = \frac{H}{2}\frac{D_1+D_2+L}{y} - D_1$$

$$I(y) = \frac{I_0 \cos^3\theta}{4\pi (D_1+D_2+L)^2} e^{-\mu X/\cos\theta} = \frac{I_0 \cos^3\theta}{4\pi (D_1+D_2+L)^2} e^{-\mu \frac{\left(\frac{H(D_1+D_2+L)}{2y} - D_1\right)}{\cos\theta}}$$

Other solutions using vertical length $y = \frac{H}{2} - y'$

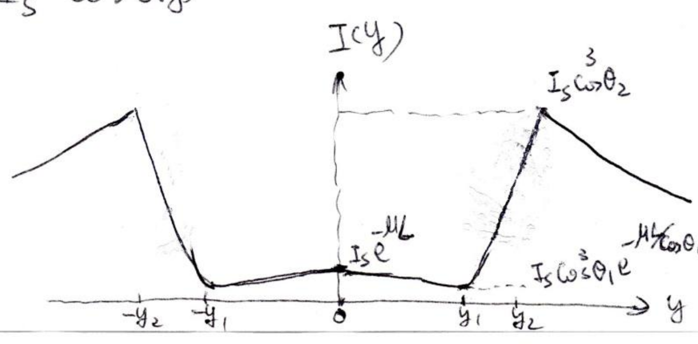
$$\tan\theta = \frac{y'}{D_1} = \frac{y}{D_1+D_2+L} \rightarrow y' = \frac{y D_1}{(D_1+D_2+L)}$$

$$I(y) = \frac{I_0 \cos^3\theta}{4\pi (D_1+D_2+L)^2} e^{-\mu y/\sin\theta} = I_s \cos^3\theta\, e^{-\mu \frac{\left(\frac{H}{2} - \frac{y D_1}{D_1+D_2+L}\right)}{\sin\theta}}$$

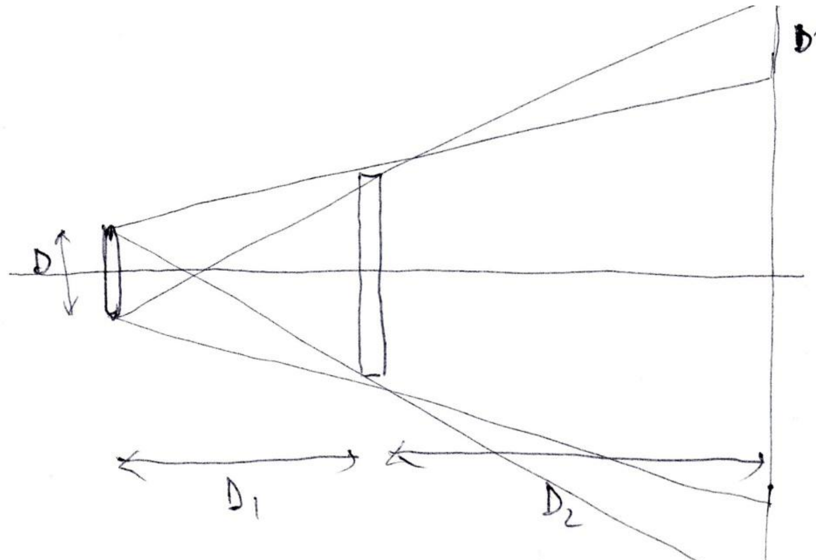
Both are equivalent.

Region III $\quad I(y) = I_s \cdot \cos^3\theta(y)$

Sketch:
Note that attenuat in Regions I & II make the signal much lower than in Region III



(b)



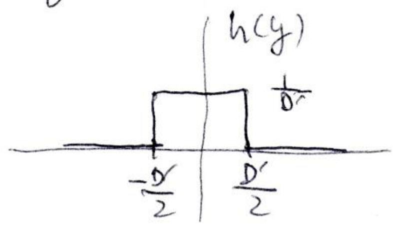
As shown above, the boundary of the detected object will become blurred with a width D'.

D' is related to D with

$$\frac{D'}{D} = \frac{D_2}{D_1} \quad \text{or} \quad D' = \frac{D_2}{D_1} D$$

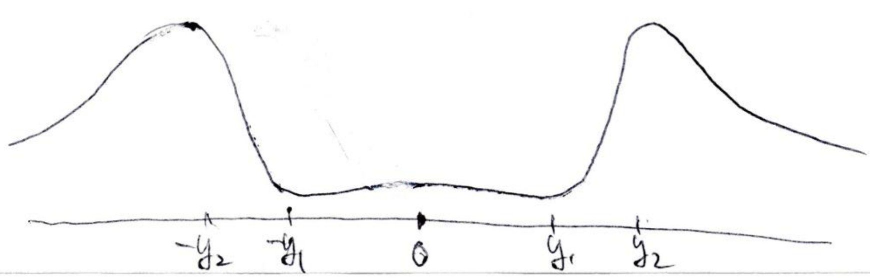
The equivalent filter response along vertical direction is

$$h(y) = \text{rect}\left(\frac{y}{D'}\right)$$



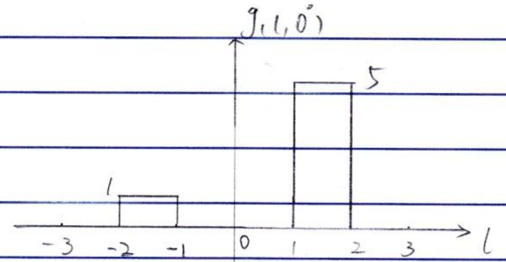
The signal captured can be approximated by

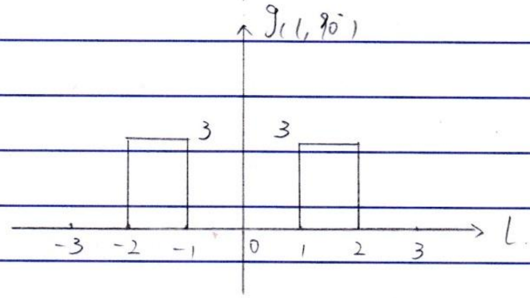
$$I'(y) = I(y) * h(y)$$



6.

(a)





(b)

0	1/6	0	0	5/6	0
3/6	4/6	3/6	3/6	8/6	3/6
0	1/6	0	0	5/6	0
0	1/6	0	0	5/6	0
3/6	4/6	3/6	3/6	8/6	3/6
0	1/6	0	0	5/6	0

(c) 135° would be a good choise
the projection of 135° will show clear that there is no object ~~at~~ in the left-top area of the slice

(d)

$g_{0°}(l) = \mathrm{rect}(l+1.5) + 5\,\mathrm{rect}\left(l-1.5\right)$

$\therefore G_{0°}(\rho) = \frac{\sin(\pi\rho)}{\pi\rho} e^{2\pi\rho\times1.5\,j} + 5\frac{\sin(\pi\rho)}{\pi\rho} e^{-2\pi\rho\times1.5\,j}$

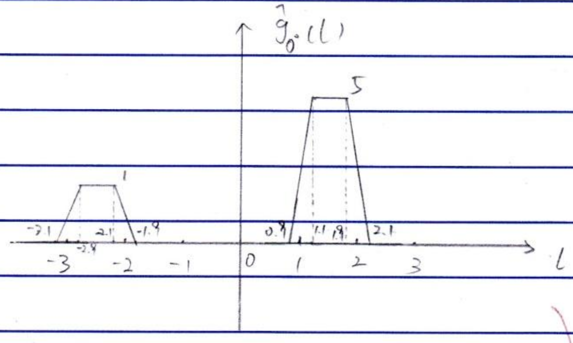
$= \frac{\sin(\pi\rho)}{\pi\rho} e^{j3\pi\rho} + 5\frac{\sin(\pi\rho)}{\pi\rho} e^{-j3\pi\rho}$

(e)

mark $\hat{g}_{0°}(l)$ is the result with area detectors.

$\hat{g}_{0°}(l) = g_{0°}(l) * h(l)$

$h(l) = 5\,\mathrm{rect}\left(\frac{l}{0.2}\right)$



(f) $\hat{f}(x,y) = f(x,y) * R^{-1}\{h(l)\} = f(x,y) * g(r)$

$\mathcal{F}\{h(l)\} = H(\rho)$ ~~= sin~~

here

$h(l) = 5\,\mathrm{rect}\left(\frac{l}{0.2}\right)$

$\therefore H(\rho) = \mathrm{sinc}(0.2\rho)$

$g(r) = H^{-1}\{\mathrm{sinc}(0.2\rho)\}$

7.

(a) $T_1 = \frac{0.693}{\lambda} \Rightarrow \lambda = \frac{0.693}{T_1}$

$A(t) = A_1 e^{-\lambda t} = A_1 e^{-t\frac{0.693}{T_1}}$

$A(\frac{T_1}{2}) = A_1 e^{-0.3465}$

~~For point B~~

$\cancel{\phi_{SPECT} = \int_{-\infty}^{3} \frac{A(x(s), y(s))}{4\pi(3-s)^2} e^{-\int_s^3 \mu(x(s'), y(s'))ds'} ds}$

~~For point B~~

$\cancel{\phi_B = \int_{-2}^{-1} \frac{A_1}{2}}$

For point B

$$\phi_B = \int_{-\infty}^{3} \frac{A(x(s), y(s))}{4\pi(3-s)^2} e^{-\int_s^3 \mu(x(s'), y(s'))ds'} ds$$

$$= \int_{-2}^{-1} \frac{A(\frac{T_1}{2})}{4\pi(3-s)^2} e^{-(-1-s)\mu_1 - \mu_3 - 2\mu_2 - \mu_3} ds$$

$$+ \int_{0}^{2} \frac{A(\frac{T_1}{2})}{4\pi(3-s)^2} e^{-(2-s)\mu_2 - \mu_3} ds$$

For point A

$$\phi_A = \int_{-\infty}^{3} \frac{A(x(s), y(s))}{4\pi(3-s)^2} e^{-\int_s^3 \mu(x(s'), y(s'))ds'} ds$$

$$= \int_{-2}^{0} \frac{A(\frac{T_1}{2})}{4\pi(3-s)^2} e^{-(0-s)\mu_2 - \mu_3 - \mu_1 - \mu_3} ds$$

$$+ \int_{1}^{2} \frac{A(\frac{T_1}{2})}{4\pi(3-s)^2} e^{-(2-s)\mu_1 - \mu_3} ds$$

where

$A(\frac{T_1}{2}) = A_1 e^{-0.3465}$

(b)

$$A\left(\frac{T_2}{2}\right) = A_2 e^{-0.3465}$$

$$\varphi = K\int_{-3}^{3} A(x(s), y(s))\,ds \cdot \exp\left(-\int_{-3}^{3} \mu(x(s'), y(s'))\,ds'\right)$$

$$= K \times A\left(\frac{T_2}{2}\right) \times (1+2) \times \exp\left(-(\mu_3+\mu_1+\mu_3+2\mu_2+\mu_3)\right)$$

$$= 3 K A\left(\frac{T_2}{2}\right) \exp\left(-(\mu_1+2\mu_2+3\mu_3)\right)$$

$= \frac{1}{\sqrt{2}} = 0.707$

(c)

$$\phi_{SPECT}(T_1) = \phi_{SPECT}\left(\frac{T_1}{2}\right) e^{-\frac{0.693}{T_1}\cdot\frac{T_1}{2}} = \phi_{SPECT}\left(\frac{T_1}{2}\right) e^{-0.3465}$$

$$\varphi_{PET}(T_1) = \varphi_{PET}\left(\frac{T_1}{2}\right) e^{-\frac{0.693}{T_2}\cdot\frac{T_2}{2}} = \varphi_{PET}\left(\frac{T_1}{2}\right) e^{-0.3465}$$