Solution to EL5823 / BE620 Final Exam (S13)

(1)

- backprojection: First do the backprojection by $b_\theta(x,y) = g(\ell,\theta)\Big|_{\ell = x\cos\theta + y\sin\theta}$

  second do the summation $\rightarrow \int_0^{\pi} b_\theta(x,y)\,d\theta = \int_0^{\pi} g(\ell,\theta)\Big|_{\ell = x\cos\theta + y\sin\theta} d\theta$

- filter backprojection: first take FT for every projection $g(\ell,\theta) \rightarrow G(\rho,\theta)$

  ~~first~~ do the filtering in frequency domain then take the inverse fourier transform: $\int_{-\infty}^{+\infty} |\rho|\, G(\rho,\theta)\, e^{-j2\pi \ell\rho} d\rho$

  second do the back projection summation $\int_0^{\pi}\left(\int_{-\infty}^{+\infty} |\rho| G(\rho,\theta) e^{-j2\pi \ell \rho} d\rho\right) d\theta \Big|_{\ell = x\cos\theta + y\sin\theta}$

- Convolution backprojection: First do the Convolution in spatial domain: $c(\ell) \underset{\text{1D}}{*} g(\ell,\theta)$

  second do the backprojection summation: $\int_0^{\pi} c(\ell) * g(\ell,\theta)\Big|_{\ell = x\cos\theta + y\sin\theta} d\theta$

- in terms of accuracy:

  backprojection summation (worse) < Convolution back projection < filter back projection (best)

we know that the backprojection summation has the worse accuracy and because in Convolution backprojection we use filter with short length the accuracy is worse than filter back projection

- in terms of Complexity:

  backprojection summation (lowest Complexity) < Convolution summation < filter back projection (highest Complexity)

we know that the back projection summation has the lowest Complexity because it needs only summation. and because in Convolution backprojection we use filter with shorter length then so its Complexity is lower than filter back projection.

(2) $f_s = \frac{1}{\tau} \geq 2W \longrightarrow \tau \leq \frac{1}{2W} \rightarrow \tau_{max} = \frac{1}{2W} = \frac{1}{4 \text{ cycles/mm}} = \frac{1}{4}$ mm

number of raws $\rightarrow M = \frac{20}{\tau_{max}} = \frac{20}{\frac{1}{4} \times 10^{-1}} = 200 \times 4 = 800$ → we need 800 samples vertically at least

number of Colourns $\rightarrow N = \frac{20}{\tau_{max}} = \frac{20}{\frac{1}{4} \times 10^{-1}} = 800$ → we need 800 " horizontally at least

the reconstructed image is M×N size $= 800 \times 800 = 640000$

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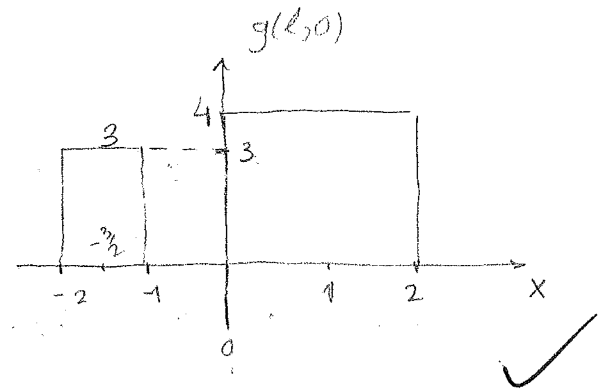
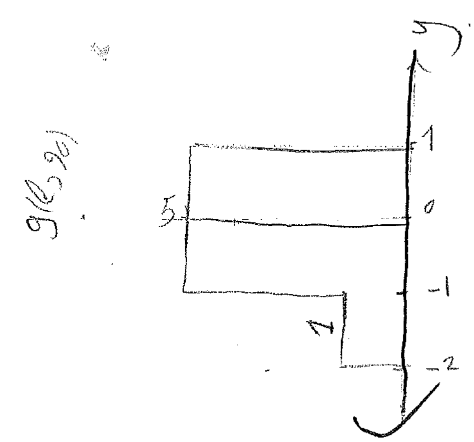
(3) a) $g(\ell, 0) \rightarrow$

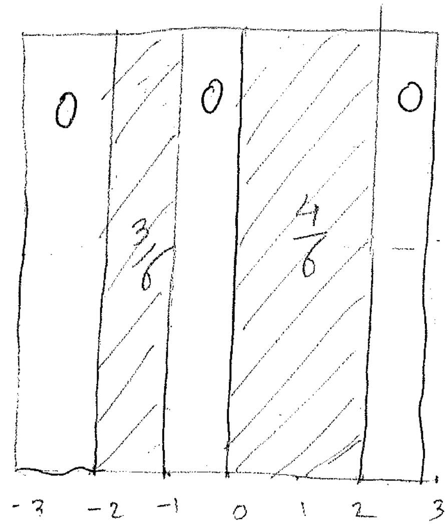


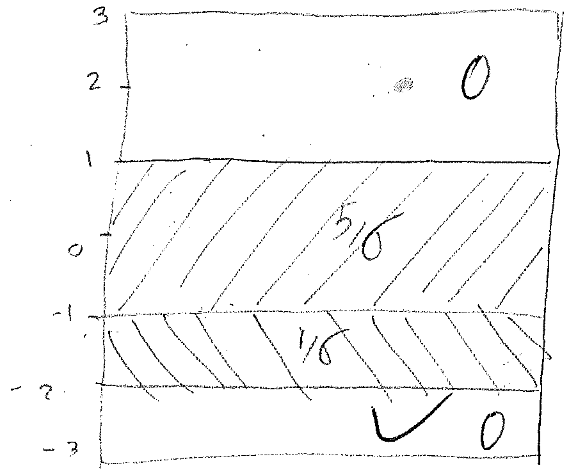
$g(\ell, 90)$



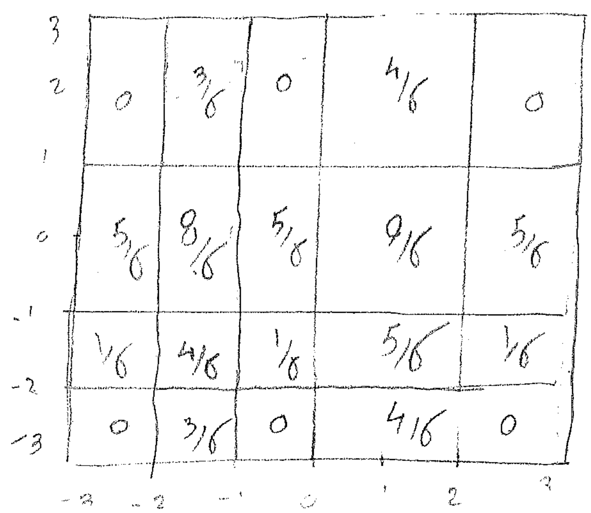
b)

$b_0(x,y)$ $\quad$ $b_{90}(x,y)$

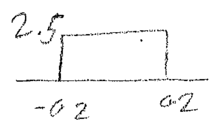




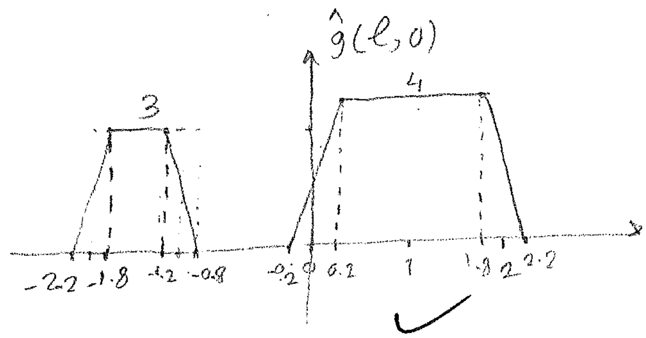
$b_0(x,y) + b_{90}(x,y)$



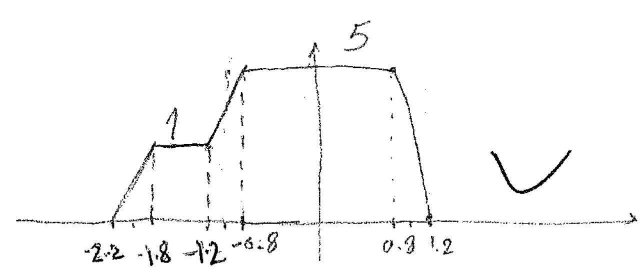
c) detector :



$\rightarrow \hat{g}(\ell,0) = g(\ell,0) \star \text{detector} \rightarrow$



$\rightarrow \hat{g}(\ell,90) = g(\ell,90) \star \text{detector} \rightarrow$



d) $g(l,0) = 3\,\mathrm{rect}\left(\frac{l+3/2}{1}\right) + 4\,\mathrm{rect}\left(\frac{l-1}{2}\right)$

$\rightarrow \mathcal{F}_{1D}\{g(l,0)\} = G(\rho,0) = 3\,\mathrm{sinc}(\rho)\,e^{j2\pi\rho\frac{3}{2}} + 8\,\mathrm{sinc}(2\rho)\,e^{-j2\pi\rho}$

$\rightarrow G(\rho,0) = \mathcal{F}_{2D}\{f(x,y)\} = F(\underbrace{\rho\cos\theta}_{u}, \underbrace{\rho\sin\theta}_{v}) = F(\rho, 0) = F(u,0)$

$\rightarrow F(u,0) = 3\,\mathrm{sinc}(u)\,e^{-j2\pi u\,3/2} + 8\,\mathrm{sinc}(2u)\,e^{-j2\pi u}$

Using FT property: $g(a(t-\tau)) \longleftrightarrow \frac{1}{a}G\left(\frac{\rho}{a}\right)e^{-j2\pi\rho\tau}$

if $g(t) \longleftrightarrow G(\rho)$

(4) a) - for SPECT we need a radiotracer which has gamma decay (isomeric decay) and also has long half-life, so we can produce the radio tracer outside of imaging facility and then bring them to imaging facility

- for PET we need a radio tracer which have positron decay, and also short half-life, so the radio tracer should be produced on site.

b)

- for SPECT we need Collimator to reject Compton scattering which reduces the image Contrast and resolution
- for PET we do not need Collimator because we use ACD (Annihilation Coincidence detector) to detect the direction of the 2 opposite direction gamma rays, so ACD by itself can reject scattering. $\rightarrow$ no need for Collimator

c) if the half life is short like PET, then radiotracer decay fast so we need to do the imaging very fast, and the radio tracer should be produced on site

if the half-life is long like SPECT, then the we do not need to do the imaging very fast, and radiotracer can be produced off site.

the desired Range of half life: Minutes to hours. $\rightarrow$ because if it is less than this range the decay is finished before we imaging and if it is longer than this it is harmful for patient.

(5) a) half life $= T_1 \rightarrow T_1 = \frac{0.693}{\lambda_1} \rightarrow \lambda_1 = \frac{0.693}{T_1}$

$$\rightarrow A(T_2) = A_1 \times e^{-\lambda_1 T_2} = A_1 \times e^{-\frac{0.693}{T_1} \times T_2}$$

$$\phi_A = \int_{-2}^{-1} \frac{A(T_2)}{4\pi(s+3)^2} e^{-(s+2)\mu_1} ds + \int_{0}^{2} \frac{A(T_2)}{4\pi(s+3)^2} e^{-\mu_1 - \mu_2 s} ds$$

$$\phi_B = \int_{-2}^{-1} \frac{A(T_2)}{4\pi(3-s)^2} e^{-(-1-s)\mu_1 - 2\mu_2} ds + \int_{0}^{2} \frac{A(T_2)}{4\pi(3-s)^2} e^{-\mu_2(2-s)} ds$$

b) half life $= T_2 \rightarrow T_2 = \frac{0.693}{\lambda_2} \rightarrow \lambda_2 = \frac{0.693}{T_2}$

$$\rightarrow A(T_3) = A_2 \times e^{-\lambda_2 T_3} = A_2 \times e^{-\frac{0.693}{T_2} T_3}$$

$$\phi_{A-B} = K \int_{-3}^{3} A(X(s), Y(s)) ds \times \exp\left\{ \int_{-3}^{3} \mu(X(s), Y(s), E) ds \right\}$$

$$= K \left[3A_2(T_3) \right] \times e^{-(\mu_1 + 2\mu_2)}.$$

Ultrasound Imaging (35 points total)

1. (~~10~~ 5 pt)
Properties of an ultrasound transducer:

i) Thickness of the piezoelectric crystal
ii) Matching layer impedance
iii) Bandwidth
iv) Aperture
v) Spacing between transducer elements

Which transducer properties affect the following characteristics (list all that are relevant)? (2 pts each)

- **a)** Axial resolution i), ii), iii)
- **b)** Lateral resolution i), iv), v)
- **c)** Attenuation i)
- **d)** Maximum steering angle v)
- **e)** Depth of field i), iv), v)

if at least two items are correct then get 1 pt.
If at least one item is wrong, deduct 1 pt

2. (20 pt).
a) (5 pt) For a clinical imaging application, you need to have a penetration depth (distance below skin / transducer) of 8-cm. What is the maximum frame rate (images per second) for a 256-line image at that depth?
[assume the sound speed, c = 1540 m/s]

Propagation length = 8x2 = 16 cm = 0.16 m
Time per line = 0.16 / 1540 ~ 104 μs
Time per image = 104 x 256 ~26,600 μs = 26.6 ms
Frame rate = 1/26.6 x 10^{-3} s ~ 37 images per second (accept 30-40 Hz)

b) (5 pt) What would be the maximum frame rate (256-line image) if the depth were only 8-mm instead of 8-cm?

Propagation length = 8x2 = 16 mm = 0.016 m
Time per line = 0.016 / 1540 ~ 10.4 μs
Time per image = 10.4 x 256 ~2,660 μs = 2.7 ms
Frame rate = 1/2.7 x 10^{-3} s ~ 370 images per second (accept 300-400 Hz)

c) (5 pt) You are given two transducers, one operating at 5-MHz and one at 50-MHz. Which transducer would you choose for application **a)**? Which one for **b)**?

a) 5-MHz; b) 50-MHz

d) (5 pt) For each of the two transducers, estimate the signal loss due to an object at a depth of 2 cm below the skin (expressed as a percentage of the transmitted amplitude).

[Assume attenuation = 1dB/cm/MHz, and recall that 20-dB is a 10x loss in signal amplitude.]

Depth 4-cm: Propagation length = 2 x 2 = 4-cm
Trans a): Attenuation = 4 x 5 = 20 dB, or 10x signal loss, i.e., 10% of transmitted signal
Trans b): Attenuation = 4x50 = 200 dB or 10^{10}x signal loss, i.e., 10^{-8} % (essentially 0).

3. (5 pt) As you scan a patient with a 5-MHz transducer, you notice there is an artery that is oriented in-line with the propagation direction of ultrasound from your transducer and that the maximum Doppler shift detected is 15-kHz. Estimate the peak blood velocity within the artery.

[Recall the Doppler equation is $f_d = 2f_0 v \cos\theta / c$, and assume the speed of sound in blood is 1600 m/s]

$\theta=0$, $\cos\theta=1$,
From Doppler equation:

$$v = cf_d/2f_0$$
$$= (1600 \text{ m/s}) \times (15 \times 10^3 \text{ /s}) / (2 \times 5 \times 10^6 \text{ /s})$$
$$= 2.4 \text{ m/s}$$