EL5823/BE6203 Medical Imaging I, Spring 2013
Final Exam, 5/20/2013, 3:00-5:30PM
(closed book, 1 sheet of notes double sided allowed)
(No peeking into the neighbors and your notes. Cheating will result in a grade of F)

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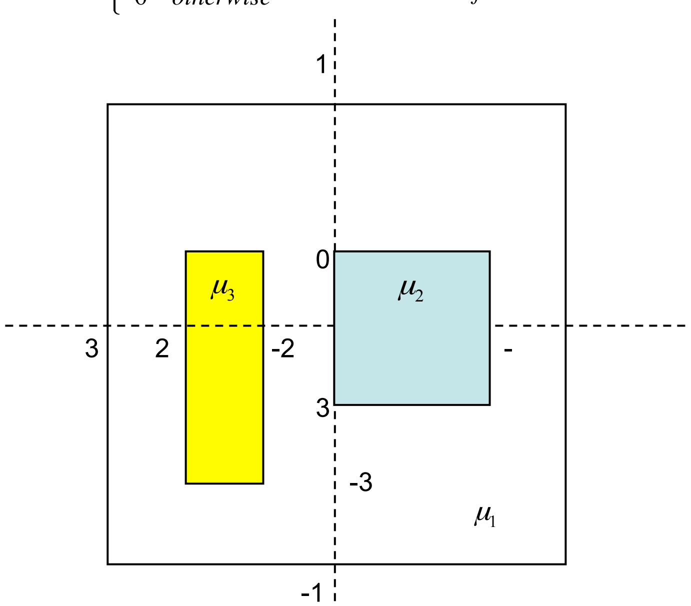
1. (20 pt) Describe the major steps in the following three reconstruction algorithms from parallel projections: backprojection, filtered backprojection, and convolution backprojection. Rank these methods in terms of accuracy and complexity, respectively. Assume that the convolution backprojection use a truncated filter with a length much shorter than the samples along each projection. Please be concise in your answer.

2. (5 pt) Suppose we know that the maximum spatial frequency of the imaged region is W=2 cycles/mm. What is the maximum spacing τ between each detector along the same projection angle? If we want to image a slice with an area of size 20cm x 20 cm, what is the minimum number of samples you need along the horizontal and vertical directions respectively? What should be your reconstructed image size (in terms of number pixels)?

3. (20 pt) Suppose the tissue slice (with dimension 6x6cm) being imaged by a parallel beam x-ray CT scanner contains distribution of the linear attenuation coefficients as shown below, with $\mu_1 = 0, \mu_2 = 2, \mu_3 = 1\ (cm^{-1})$. (a) Assume the detector is a point detector. Sketch the projection $g(l,\theta)$ as a function of l, for θ=0 and 90 degrees, respectively. You should indicate the magnitudes of the projected values where necessary on your sketch. Also clearly specify any transition points in the l-axis. (b) Sketch the image obtained by backprojections from both 0 and 90 degree projections. You should assume that you know the dimension of the tissue being imaged and normalize your backprojection using the known dimensions. (c) What will be the projected functions for θ=0 and 90 degree, respectively, if the detector is an area detector with width 0.4 cm. Sketch the projection functions. (d) Determine the Fourier transform of the original image along a line with orientation θ=0 degree in the frequency domain. You can express your solution in terms of G(f;W) below.

   Hint: The Fourier transform of the rectangular function is given by:

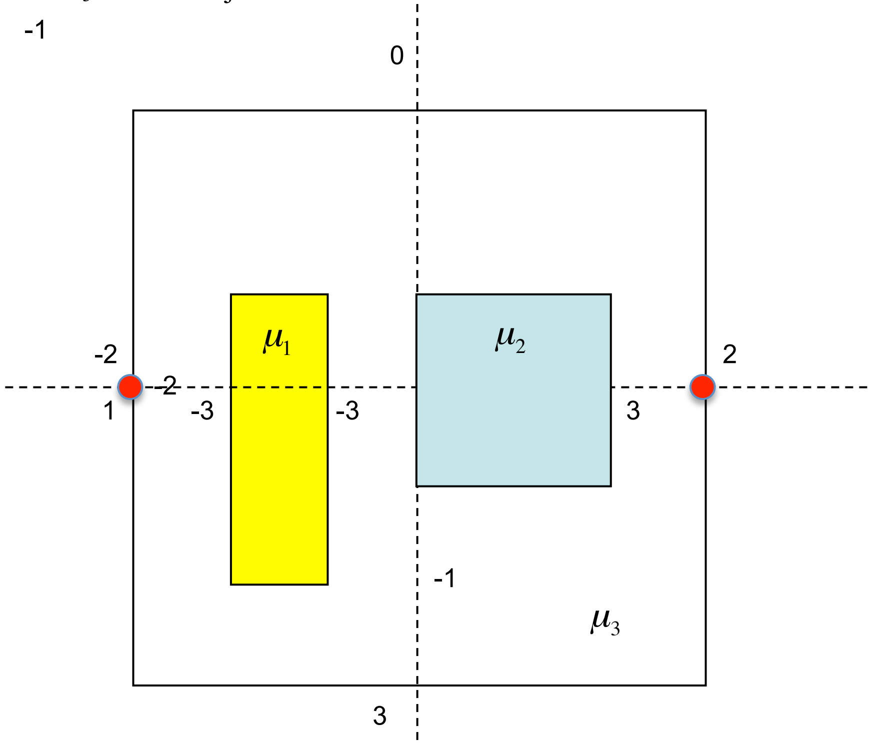
$$g(x;W) = \begin{cases} 1 & |x| < W/2 \\ 0 & otherwise \end{cases} \Leftrightarrow G(f;W) = \frac{\sin \pi W f}{\pi f}$$



4. (10 pt) (a) In nuclear imaging, what type of radiotracer is needed for SPECT? What type is needed for PET? (b) Do we need to use a collimator in SPECT camera? What about for PET? Why? (c) How does the half-life of the radio tracer affect the imaging time after the patient is injected the radio tracer?

5. (15 pt) A 2-D slice to be imaged is shown below, which consists of two regions with linear attenuation coefficients μ_1 and μ_2. The background has a linear attenuation coefficient of $\mu_3 = 0$.

   a. Suppose a solution containing a gamma ray emitting radio tracer with an initial radioactivity of A_1 and half life time of T_1 is injected into both regions. We image the radioactivity distribution in this slice using a rotating SPECT camera. Compute the measured signal by the camera at positions A and B, respectively at time T_2 after the injection of the radionuclide solution.
   b. Now suppose the radio tracer in (a) is replaced by a positron emitting radio tracer with the initial radioactivity A_2 and half-life T_2. This time the slice is imaged using a PET scanner. Compute the measured signal by the pair of cameras positioned at A and B, at time T_3 after the injection of the radionuclide solution.



6. (5 pt) Some properties of an ultrasound transducer array are listed below:

i) Thickness of the piezoelectric crystal
ii) Matching layer impedance
iii) Bandwidth
iv) Aperture
v) Spacing between transducer elements

Which transducer properties affect the following characteristics (list all that are relevant)? (1 pt each)

a) Axial resolution
b) Lateral resolution
c) Attenuation
d) Maximum steering angle
e) Depth of field

7. (20 pt). **a)** (5 pt) For a clinical imaging application, you need to have a penetration depth (distance below skin / transducer) of 8-cm. What is the maximum frame rate (images per second) for a 256-line image at that depth? [assume the sound speed, c = 1540 m/s]
b) (5 pt) What would be the maximum frame rate (256-line image) if the depth were only 8-mm instead of 8-cm?
c) (5 pt) You are given two transducers, one operating at 5-MHz and one at 50-MHz. Which transducer would you choose for application **a)**? Which one for **b)**?
d) (5 pt) For each of the two transducers, estimate the signal loss due to an object at a depth of 2 cm below the skin (expressed as a percentage of the transmitted amplitude).
[Assume attenuation = 1dB/cm/MHz, and recall that 20-dB is a 10x loss in signal amplitude.]

8. (5 pt) As you scan a patient with a 5-MHz transducer, you notice there is an artery that is oriented in-line with the propagation direction of ultrasound from your transducer and that the maximum Doppler shift detected is 15-kHz. Estimate the peak blood velocity within the artery.

[Recall the Doppler equation is $f_d = 2f_0 v \cos\theta / c$, and assume the speed of sound in blood is 1600 m/s]