EL5123 --- Image Processing

Median Filtering and Morphological Filtering

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With contribution from Zhu Liu, Onur Guleryuz, and Gonzalez/Woods, Digital Image Processing, 2ed

Lecture Outline

- Median filter
- Rank order filter
- Bilevel Morphological filters
  - Dilation and erosion
  - Opening and closing
- Grayscale Morphological filters

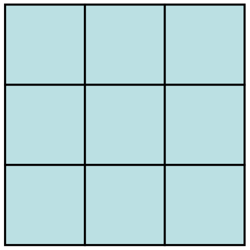
Median Filter

- Problem with Averaging Filter
  - Blur edges and details in an image
  - Not effective for impulse noise (Salt-and-pepper)
- Median filter:
  - Taking the median value instead of the average or weighted average of pixels in the window
    - Sort all the pixels in an increasing order, take the middle one
  - The window shape does not need to be a square
  - Special shapes can preserve line structures

Median Filter: 3x3 Square Window

100	100	100	100	100
100	200	205	203	100
100	195	200	200	100
100	200	205	195	100
100	100	100	100	100

Window shape



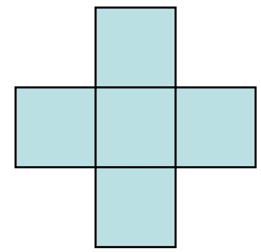
100	100	100	100	100
100	100	200	100	100
100	200	200	200	100
100	100	195	100	100
100	100	100	100	100

Matlab command: medfilt2(A,[3 3])

Median Filter: 3x3 Cross Window

100	100	100	100	100
100	200	205	203	100
100	195	200	200	100
100	200	205	195	100
100	100	100	100	100

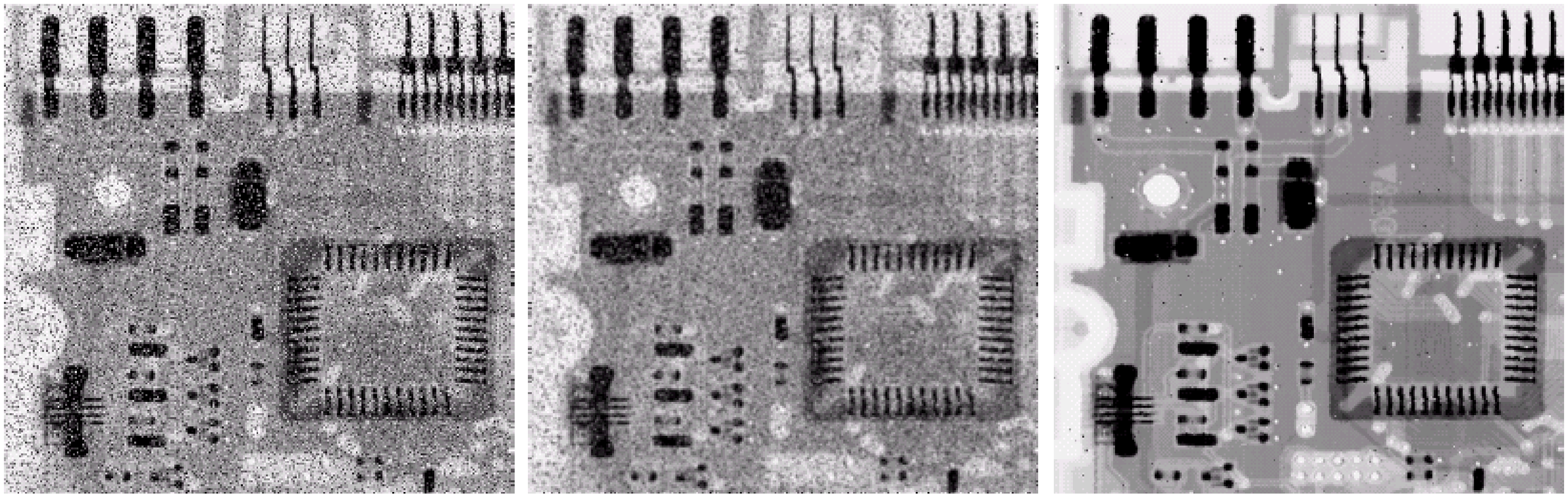
Window shape



100	100	100	100	100
100	195	200	200	100
100	200	200	200	100
100	195	200	195	100
100	100	100	100	100

Note that the edges of the center square are better reserved

Example



a b c

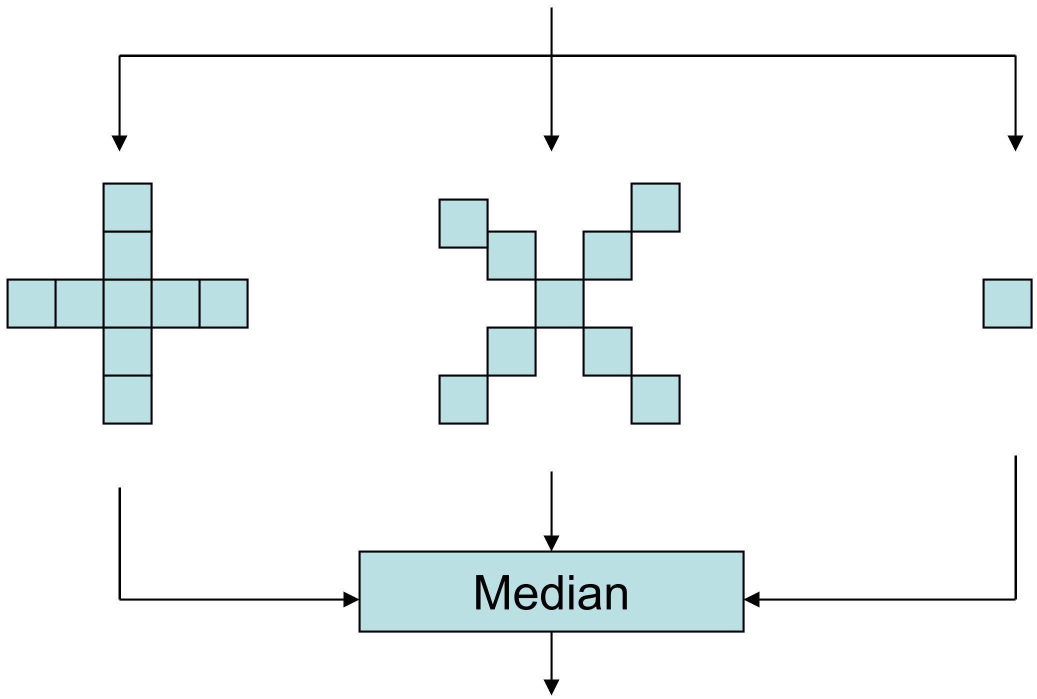
FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3 × 3 averaging mask. (c) Noise reduction with a 3 × 3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Rank order filters

- Rank order filters
  - Instead of taking the mean, rank all pixel values in the window, take the n-th order value.
  - E.g. max or min or median
- Properties
  - Non-linear $T(f_1 + f_2) \neq T(f_1) + T(f_2)$

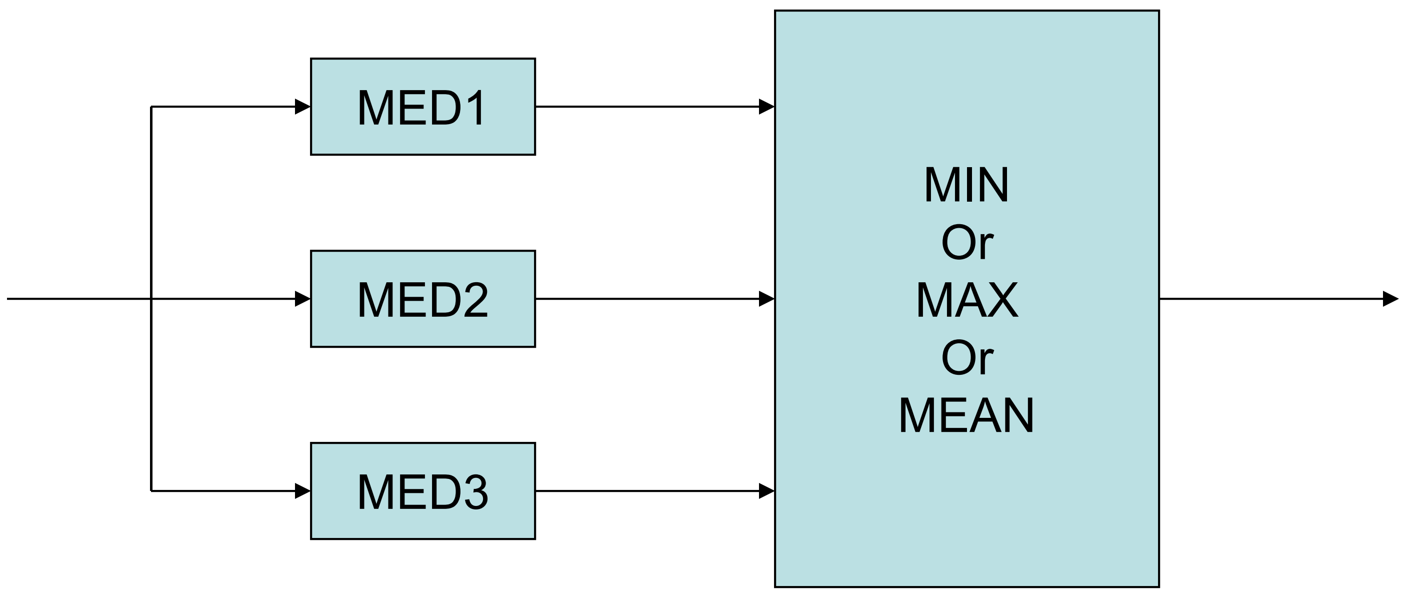
Multi-level Median Filtering

- To reduce the computation, one can concatenate several small median filters to realize a large window operation.
- When the small windows are designed properly, this approach can also help reserve edges better.



Hybrid Linear/Median Filter

- One can combine median filters with linear or rank order filters.

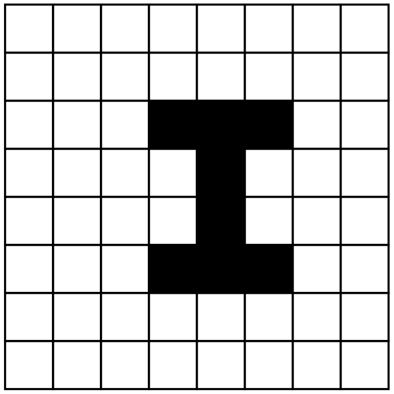


Morphological Processing

- Morphological operations are originally developed for bilevel images for shape and structural manipulations.
- Basic functions are *dilation* and *erosion*.
- Concatenation of dilation and erosion in different orders result in more high level operations, including *closing* and *opening*.
- Morphological operations can be used for smoothing or edge detection or extraction of other features.
- Belongs to the category of spatial domain filter.

Morphological Filters for Bilevel Images

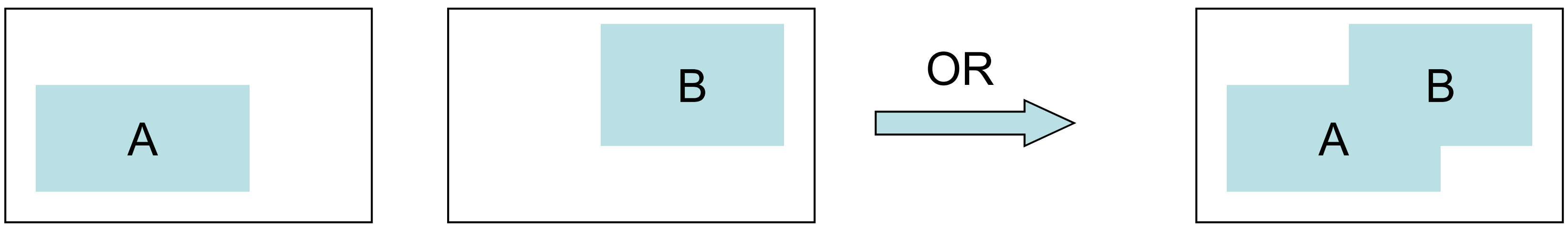
- A binary image can be considered as a set by considering "black" pixels (with image value "1") as elements in the set and "white" pixels (with value "0") as outside the set.



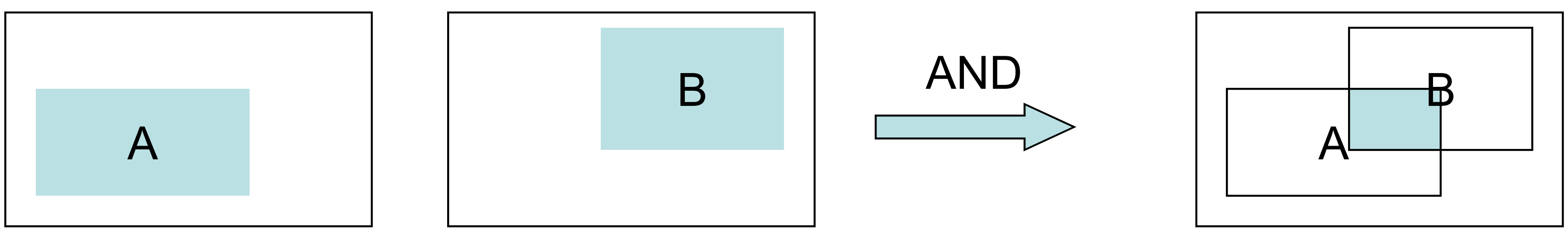
- Morphological filters are essentially set operations

Basic Set Operations

- Let x, y, z, … represent locations of 2D pixels, e.g. $x = (x_1, x_2)$, S denote the complete set of all pixels in an image, let A, B, … represent subsets of S.
- Union (OR) $A \cup B = \{x : x \in A \; or \; x \in B\}$



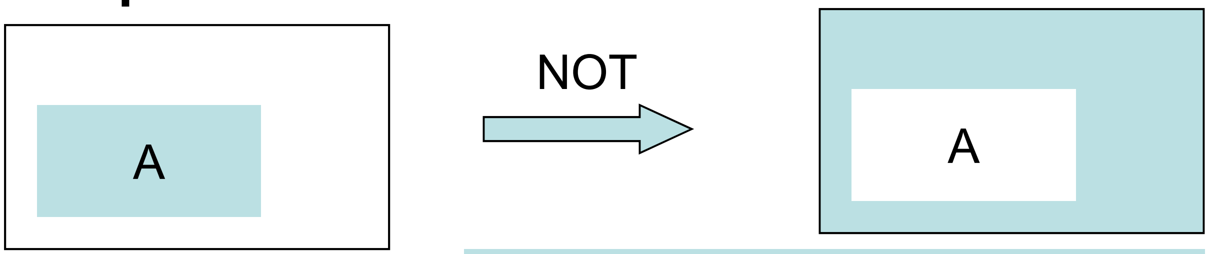
- Intersection (AND) $A \cap B = \{x : x \in A \; and \; x \in B\}$



Basic Set Operations

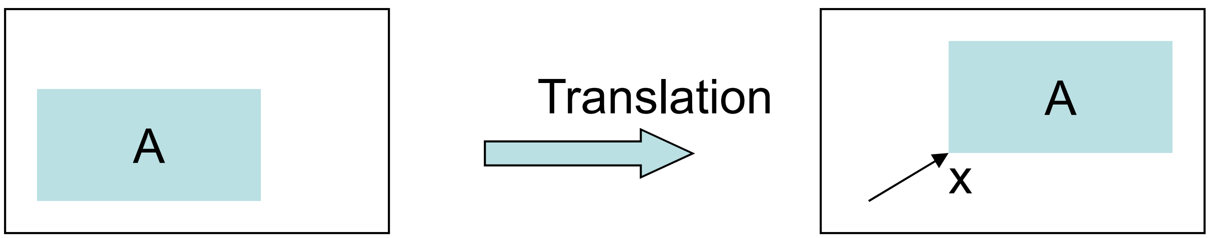
- Complement

$$\overline{A} = \{x : x \in S \ and \ x \notin A\}$$



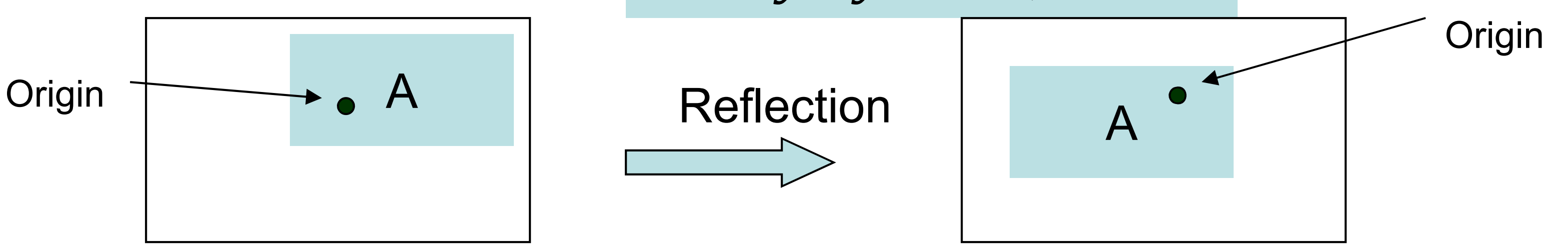
- Translation

$$(A)_x = \{z : z = y + x, \ y \in A\}$$



- Reflection

$$\hat{A} = \{y : y = -x, \ x \in A\}$$



Dilation

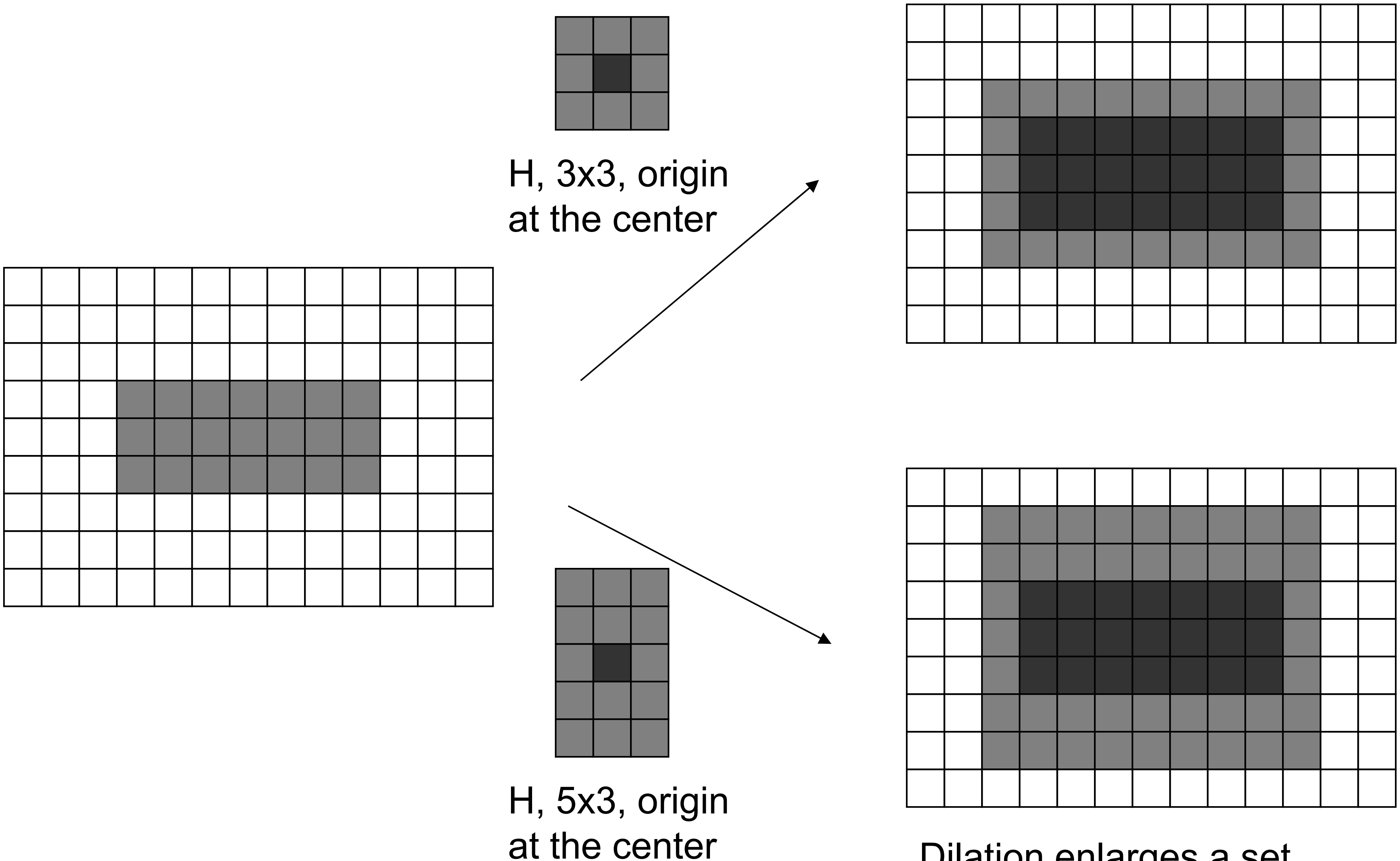
- Dilation of set F with a structuring element H is represented by $F \oplus H$

$$F \oplus H = \{x : (\hat{H})_x \cap F \neq \Phi\}$$

where Φ represent the empty set.
- $G = F \oplus H$ is composed of all the points that when Ĥ shifts its origin to these points, at least one point of Ĥ is included in F.
- If the origin of H takes value "1",

$$F \subset F \oplus H$$

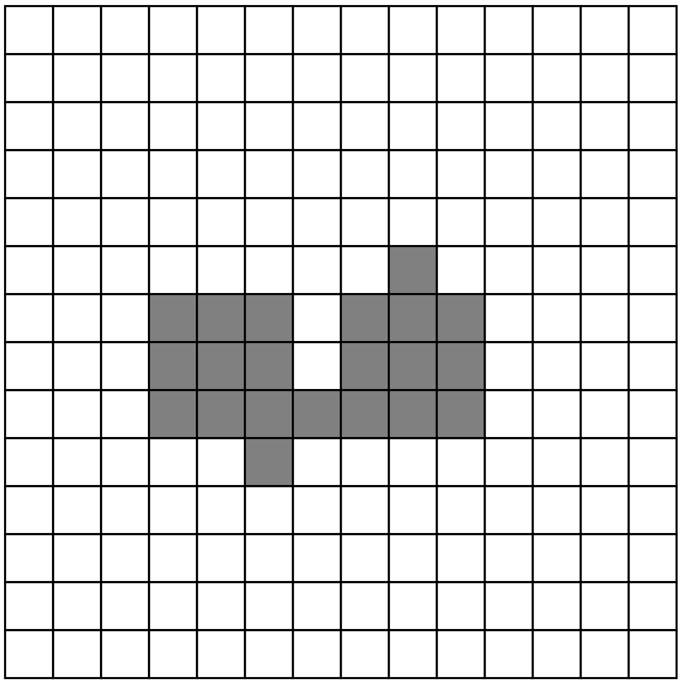
Example of Dilation (1)



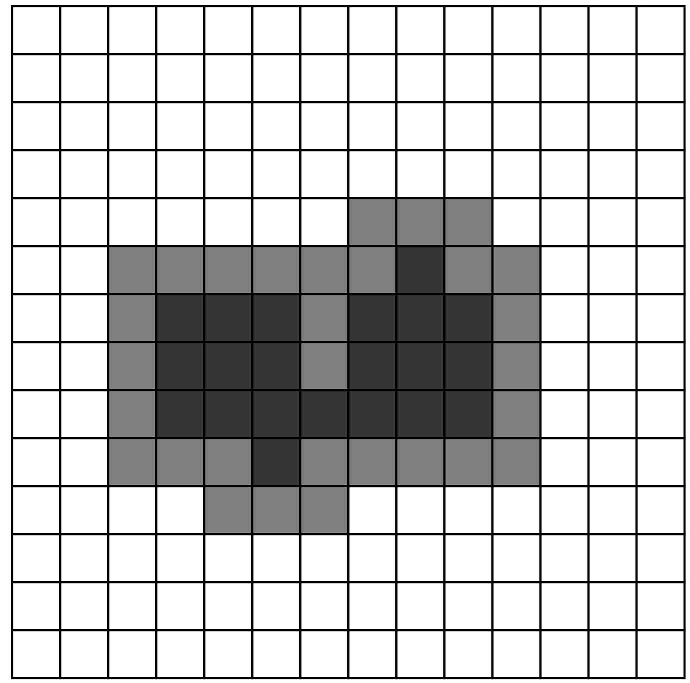
Dilation enlarges a set.

Example of Dilation (2)

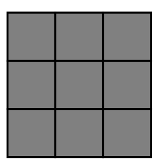
Note that the narrow ridge is closed



F



G



H, 3x3, origin at the center

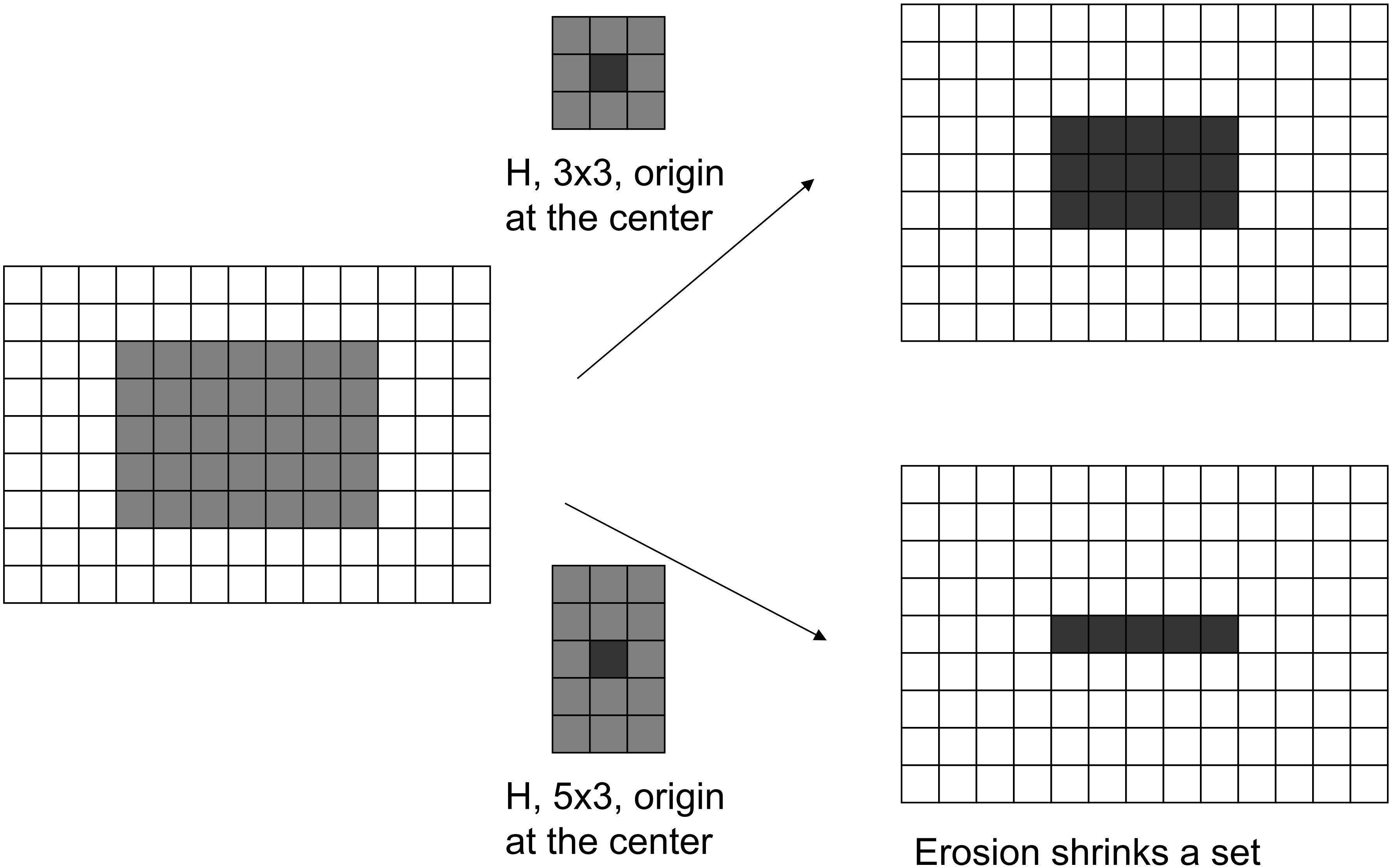
Erosion

- Erosion of set F with a structuring element H is represented by $F \Theta H$, and is defined as,

$$F \Theta H = \{x : (H)_x \subset F\}$$

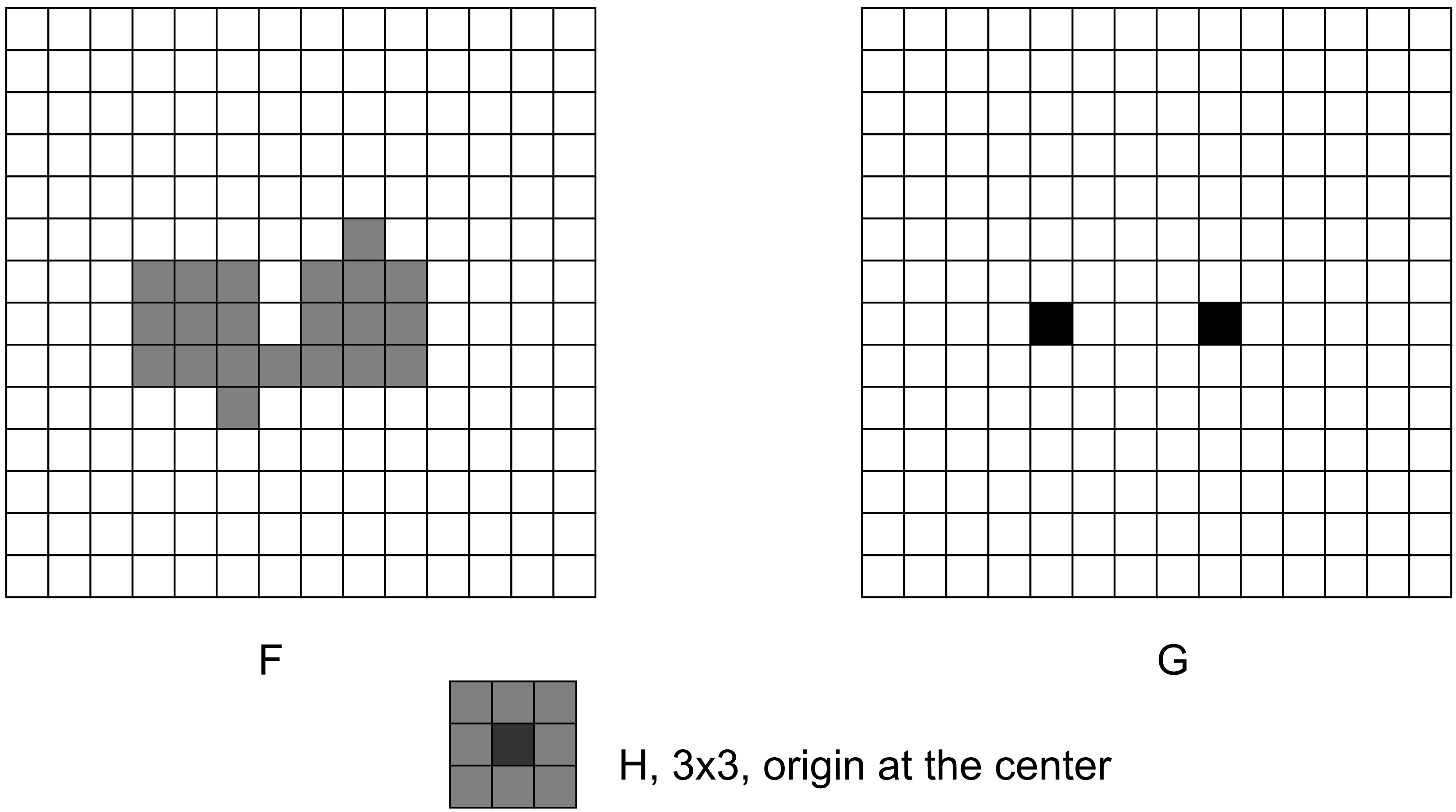
- $G = F \Theta H$ is composed of points that when H is translated to these points, every point of H is contained in F.
- If the origin of H takes value of "1", $F \Theta H \subset F$

Example of Erosion (1)



Erosion shrinks a set

Example of Erosion (2)



Structuring element

- The shape, size, and orientation of the structuring element depend on application.
- A symmetrical one will enlarge or shrink the original set in all directions.
- A vertical one, will only expand or shrink the original set in the vertical direction.

Properties of Dilation and Erosion Operators

- Communitivity

$$A \oplus B = B \oplus A, \quad but \ A \Theta B \neq B \Theta A.$$

$$\begin{aligned} A \oplus B &= \{x : (\hat{B})_x \cap A \neq \Phi\} = \{x : \exists y, y \in A, y \in (\hat{B})_x\} \\ &= \{x : \exists y, y \in A, y - x \in \hat{B}\} = \{x : \exists y, y \in A, x - y \in B\} \\ &= \{x : \exists z, x - z \in A, z \in B\} = \{x : \exists z, z - x \in \hat{A}, z \in B\} \\ &= \{x : \exists z, z \in (\hat{A})_x, z \in B\} = \{x : (\hat{A})_x \cap B \neq \Phi\} = B \oplus A \end{aligned}$$

- Duality

$$\overline{A \Theta B} = \overline{A} \oplus \hat{B}, \quad \overline{A \oplus B} = \overline{A} \Theta \hat{B}.$$

Properties of Dilation and Erosion Operators

- Distributivity

$$A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C),$$
$$A \ominus (B \cup C) = (A \ominus B) \cap (A \ominus C),$$
$$(A \cap B) \ominus C = (A \ominus C) \cap (B \ominus C).$$

- Chain rule

$$(A \oplus B) \oplus C = A \oplus (B \oplus C),$$
$$(A \ominus B) \ominus C = A \ominus (B \oplus C).$$

Closing and Opening

- Closing

$$F \bullet H = (F \oplus H)\Theta H$$

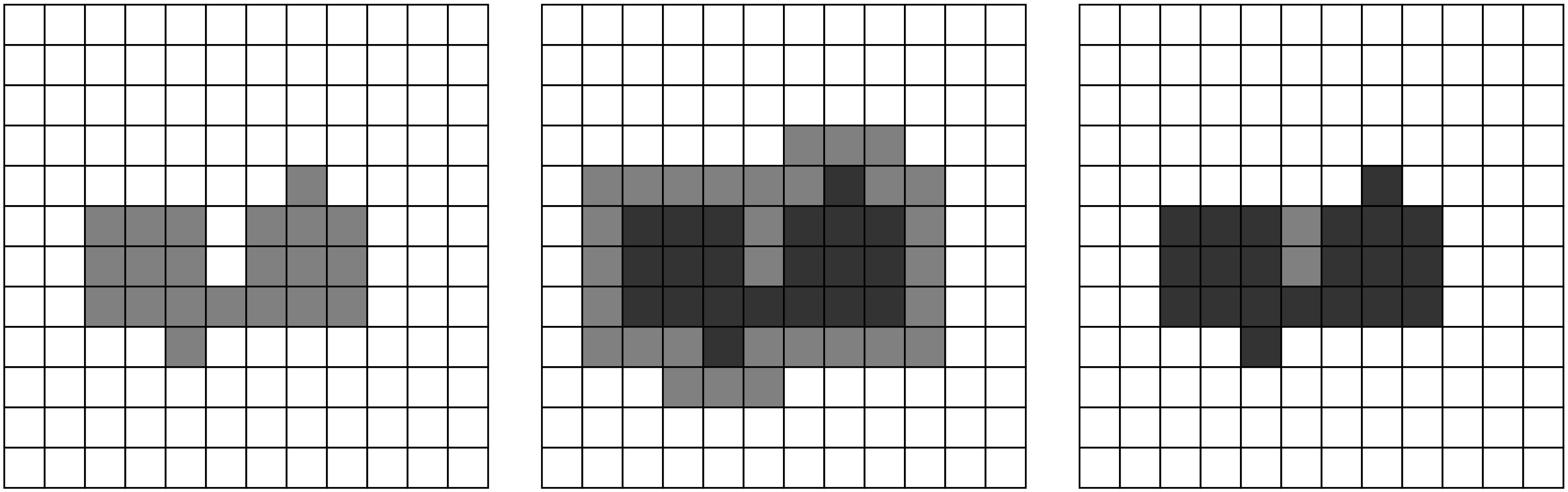
  – Smooth the contour of an image
  – Fill small gaps and holes

- Opening

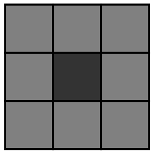
$$F \circ H = (F\Theta H) \oplus H$$

  – Smooth the contour of an image
  – Eliminate false touching, thin ridges and branches

Example of Closing

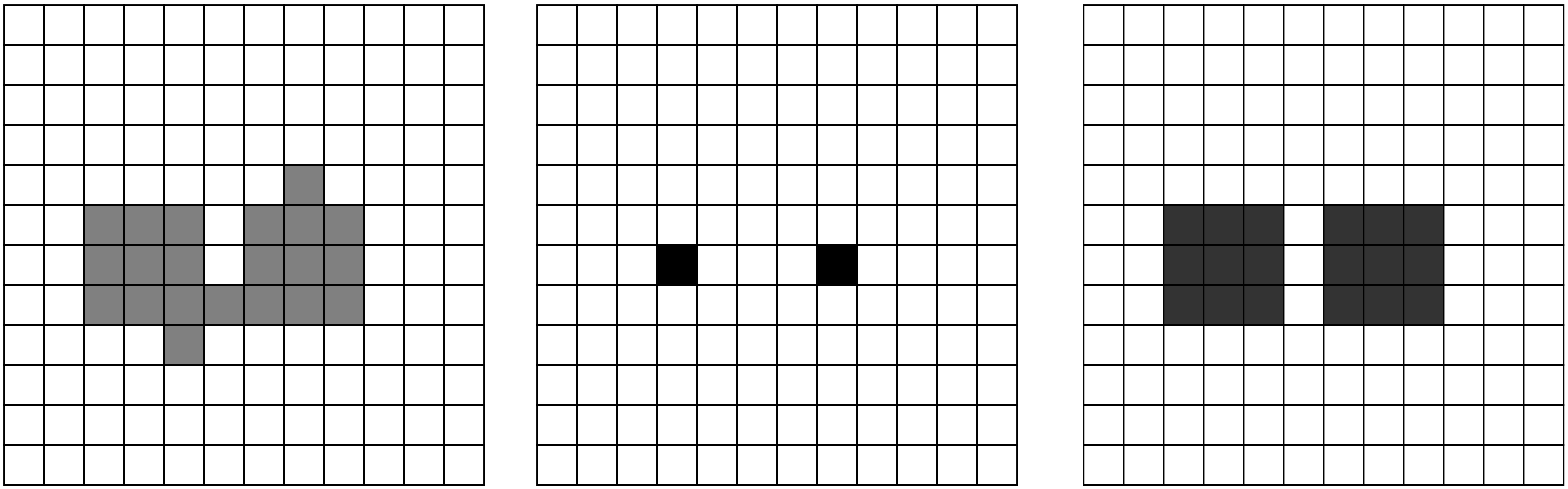


F $F \oplus H$ $(F \oplus H)\Theta H$



H, 3x3, origin at the center

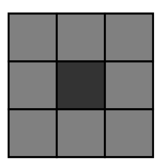
Example of Opening



F

$F \ominus H$

$(F \ominus H) \oplus H$



H, 3x3, origin at the center

Example of Opening and Closing

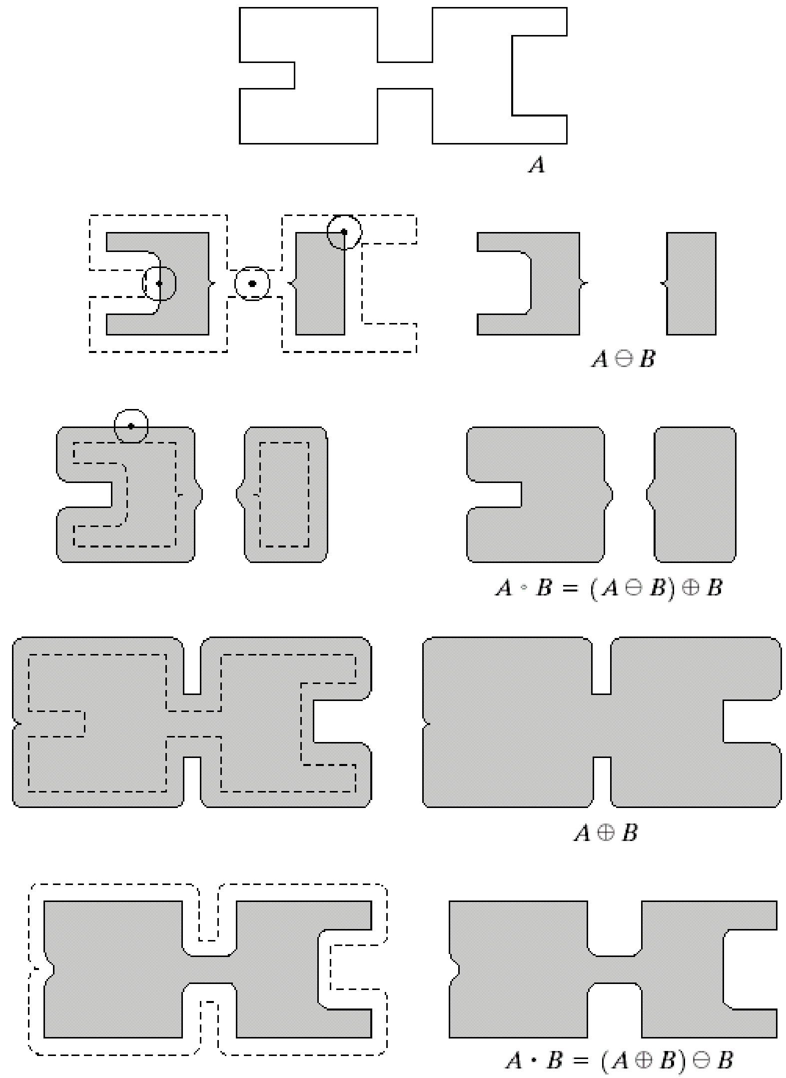


FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.

Properties of Opening and Closing Operators

- Duality

$$\overline{F \bullet H} = \overline{F} \circ \hat{H}, \quad \overline{F \circ H} = \overline{F} \bullet \hat{H}.$$

- Idempotence

$$(F \bullet H) \bullet H = F \bullet H, \quad (F \circ H) \circ H = F \circ H.$$

- Smoothing
  - Removal of small holes and narrow branches can be accomplished by concatenating opening with closing: $G = (F \circ H) \bullet H$

Morphological Filters for Grayscale Images

- The structure element h is a 2D grayscale image with a finite domain (D_h), similar to a filter
- The morphological operations can be defined for both continuous and discrete images.

Dilation for Grayscale Image

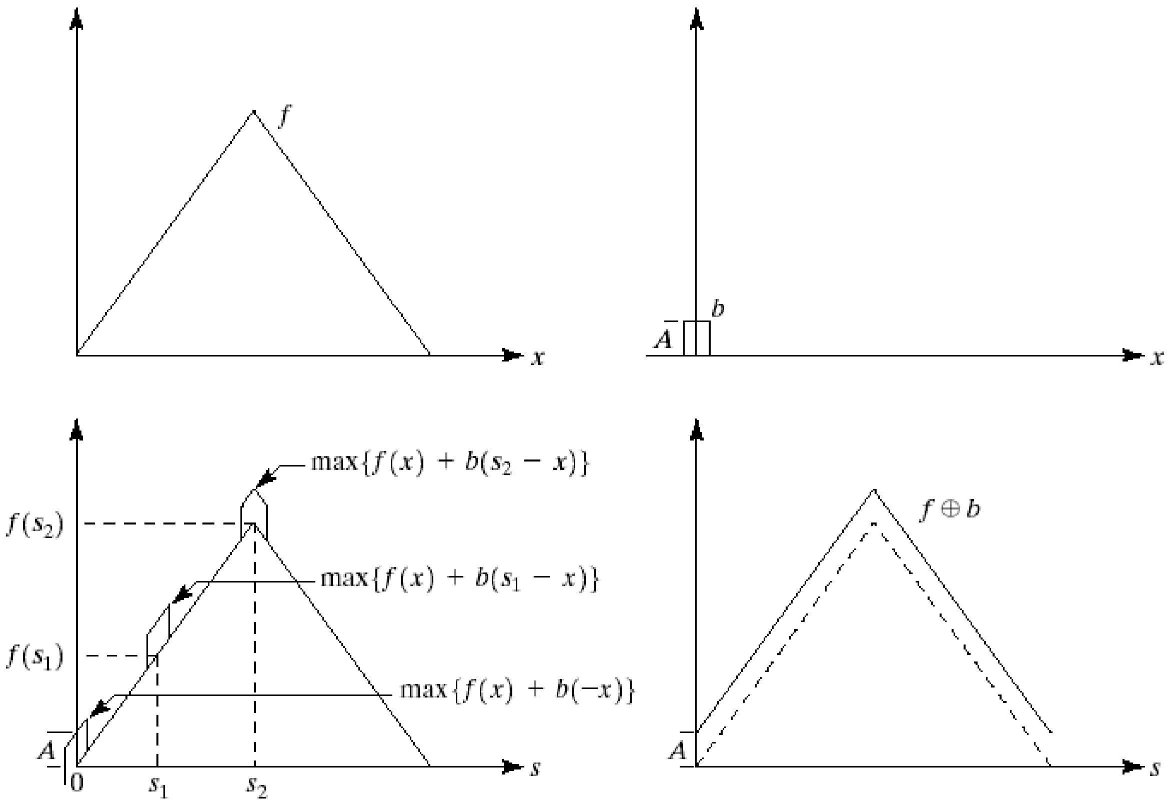
- Dilation

$$(f \oplus h)(x,y) = \max\{f(x-s,y-t)+h(s,t);(s,t)\in D_h,(x-s,y-t)\in D_f\}$$

  - Similar to linear convolution, with the max operation replacing the sums of convolution and the addition replacing the products of convolution.
  - The dilation chooses the maximum value of f+h in a neighborhood of f defined by the domain of h.
  - If all values of h are positive, then the output image tends to be brighter than the input, dark details (e.g. dark dots/lines in a white background) are either reduced or eliminated.

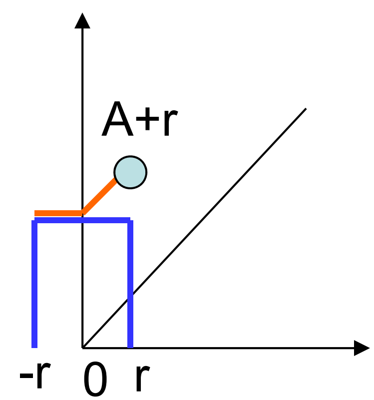
Illustration of 1-D Grayscale Dilation

$$(f \oplus b)(s) = \max\{f(s-x)+b(x);\, x \in D_b\} = \max\{f(x)+b(s-x);\, s-x \in D_b\}$$



$D_b = [-r, r]$

$s - r \le x \le s + r$



Wrong result!

a b
c d

FIGURE 9.27 (a) A simple function. (b) Structuring element of height A. (c) Result of dilation for various positions of sliding b past f. (d) Complete result of dilation (shown solid).

- Show correct result (in Red!)

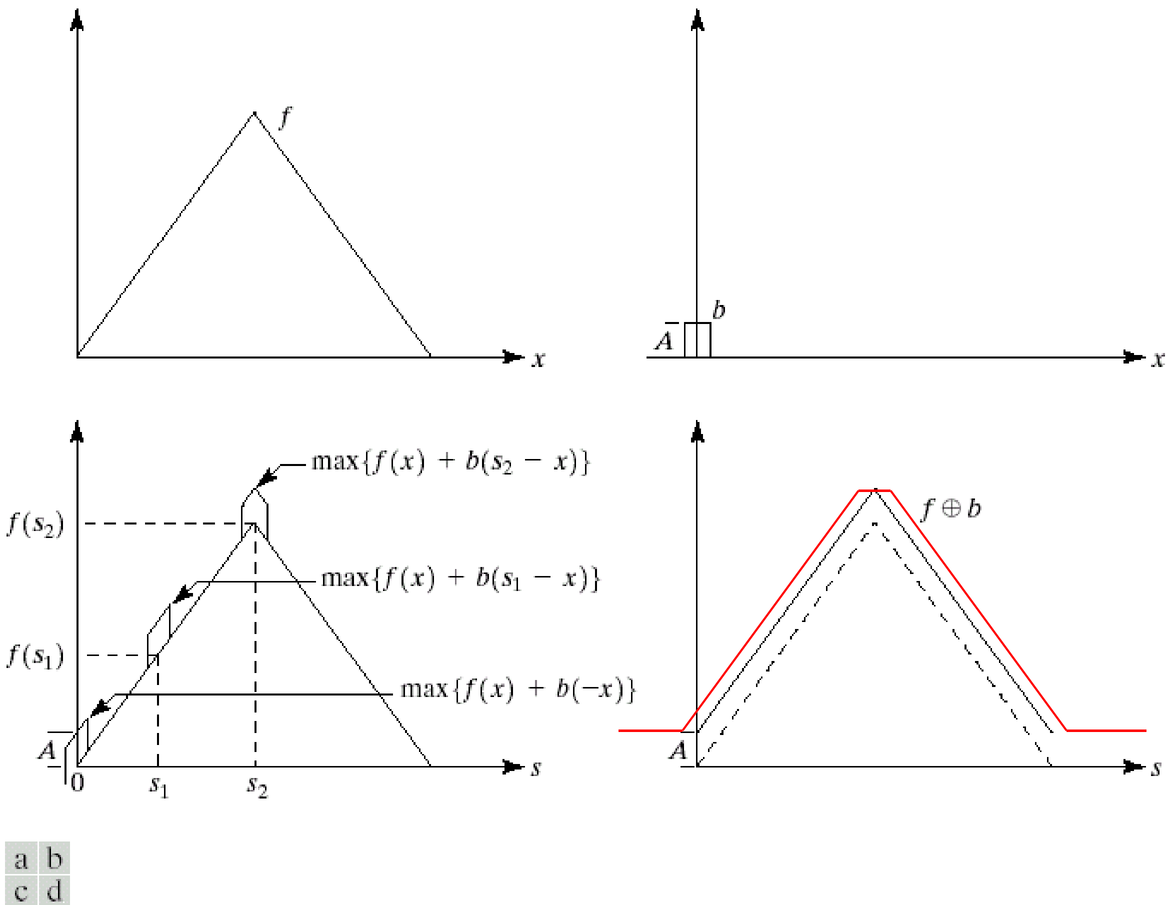


FIGURE 9.27 (a) A simple function. (b) Structuring element of height A. (c) Result of dilation for various positions of sliding b past f. (d) Complete result of dilation (shown solid).

Erosion for Grayscale Image

- Erosion

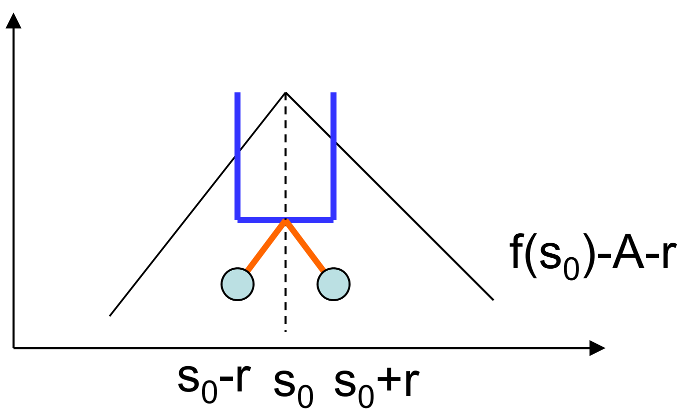
$$(f \Theta h)(x,y) = \min\{f(x+s, y+t) - h(s,t); (s,t) \in D_h, (x+s, y+t) \in D_f\}$$

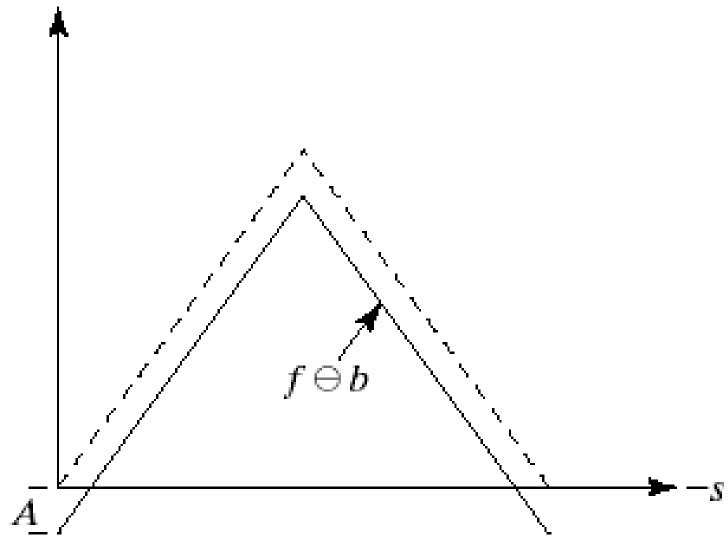
  - Similar to linear correlation, with the min operation replacing the sums of correlation and the subtraction replacing the products of correlation.
  - The erosion chooses the minimum value of f-h in a neighborhood of f defined by the domain of h.
  - If all values of h are positive, then the output image tends to be darker than the input, brighter details (e.g. white dots/lines in a dark background) are either reduced or eliminated.

Illustration of 1-D Grayscale Erosion

$$(f \Theta h)(s) = \min\{f(s+x) - b(x); x \in D_b\}$$

FIGURE 9.28
Erosion of the function shown in Fig. 9.27(a) by the structuring element shown in Fig. 9.27(b).

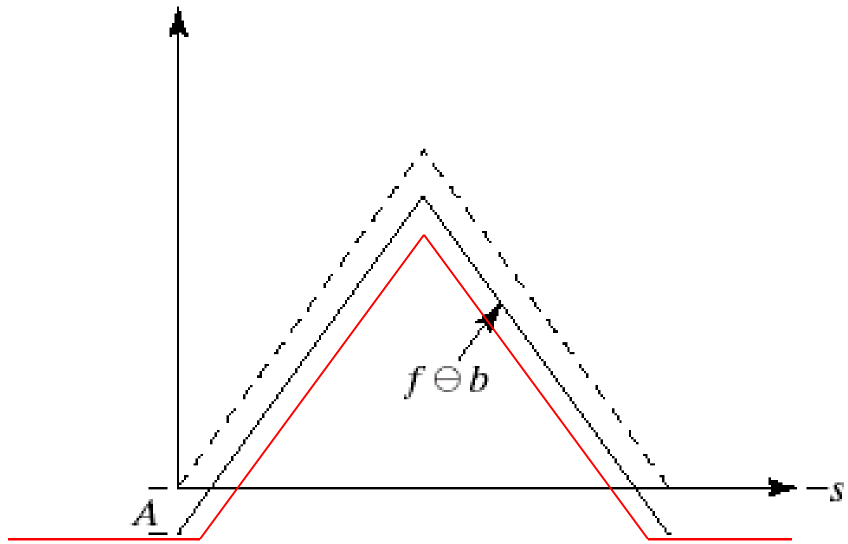




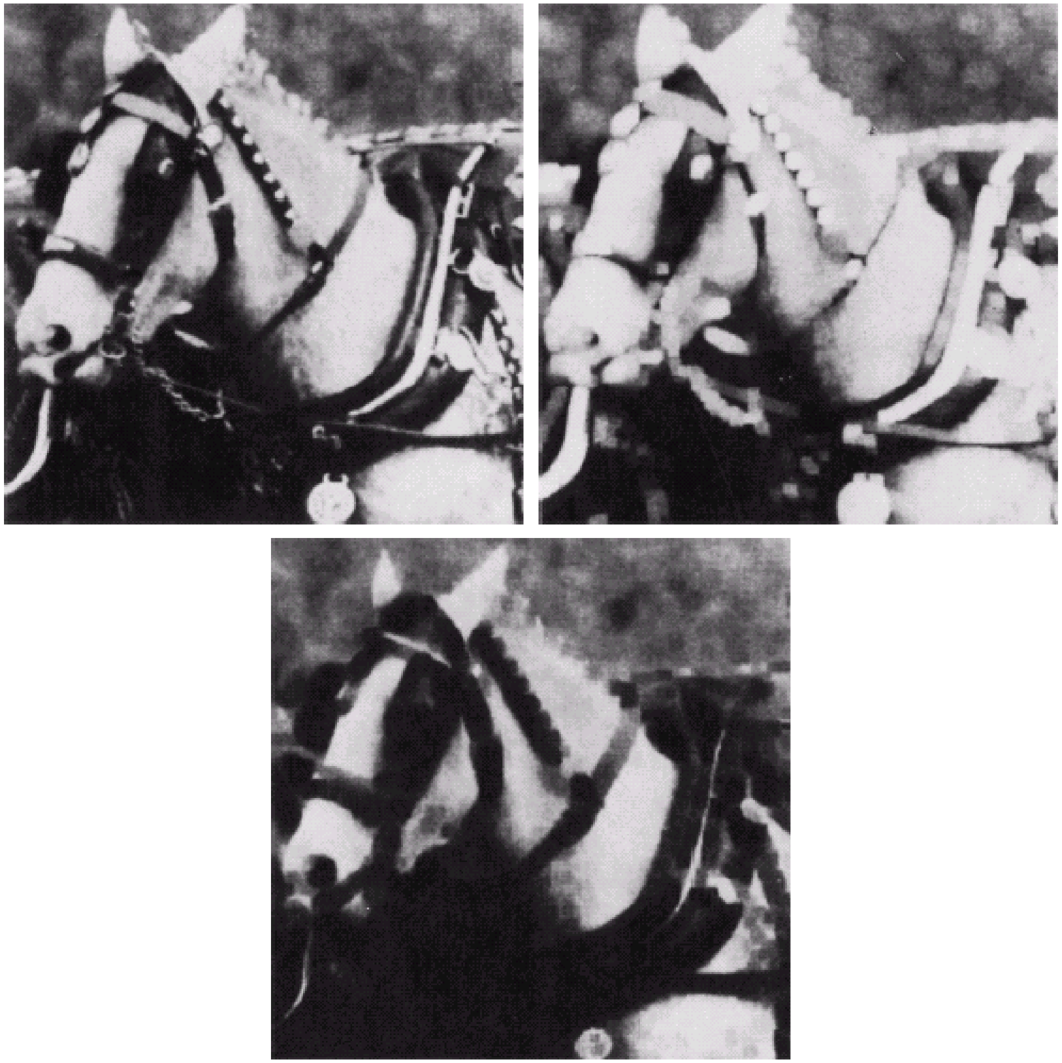
Wrong result!

- Show correct result

FIGURE 9.28
Erosion of the function shown in Fig. 9.27(a) by the structuring element shown in Fig. 9.27(b).



Example of Grayscale Dilation and Erosion



a b
c

FIGURE 9.29 (a) Original image. (b) Result of dilation. (c) Result of erosion. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Opening for Grayscale Image

- Opening

$$f \circ h = (f \ominus h) \oplus h$$

- Geometric interpretation
  - Think f as a surface where the height of each point is determined by its gray level.
  - Think the structure element has a gray scale distribution as a half sphere.
  - Opening is the surface formed by the highest points reached by the sphere as it rolls over the entire surface of f from underneath.

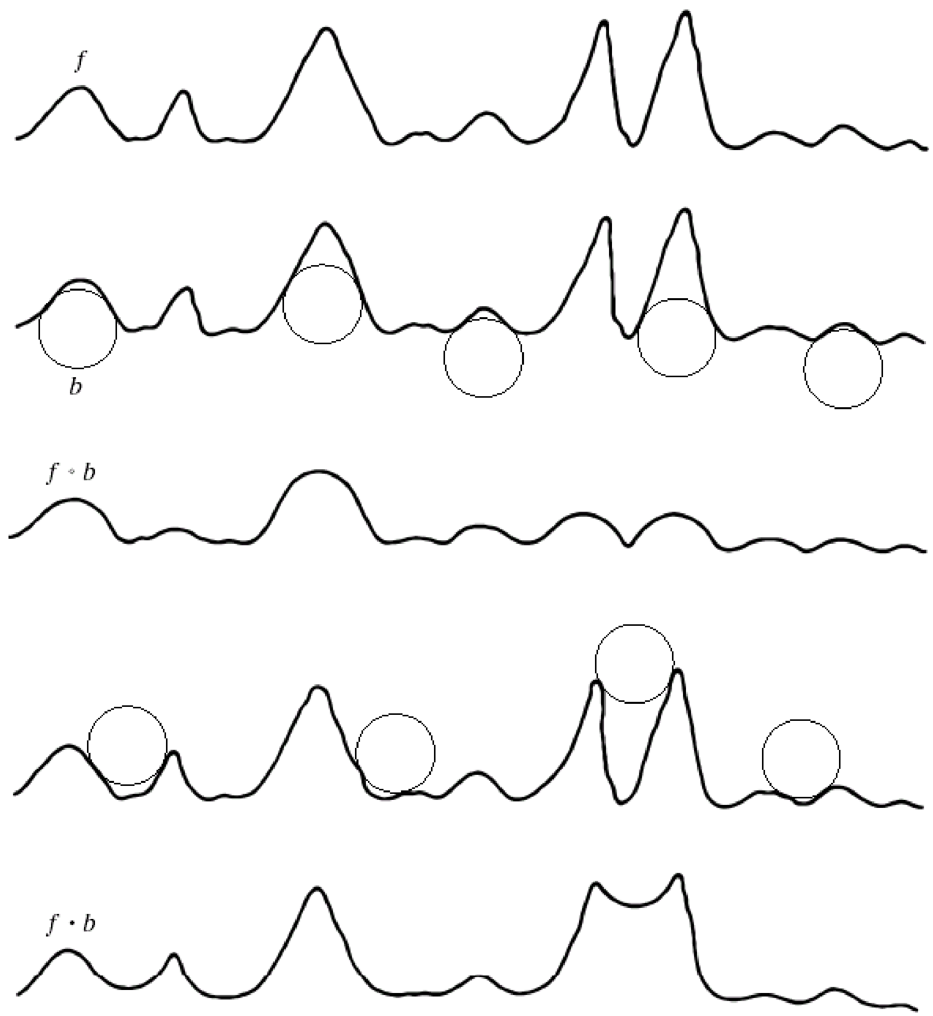
Closing for Grayscale Image

- Closing

$$f \bullet h = (f \oplus h) \Theta h$$

- Geometric interpretation
  - Think f as a surface where the height of each point is determined by its gray level.
  - Think the structure element has a gray scale distribution as a half sphere.
  - Closing is the surface formed by the lowest points reached by the sphere as it slides over the surface of f from above.

Illustration of 1-D Grayscale Opening and Closing



a
b
c
d
e

FIGURE 9.30 (a) A gray-scale scan line. (b) Positions of rolling ball for opening. (c) Result of opening. (d) Positions of rolling ball for closing. (e) Result of closing.

Opening

Eliminate false touching, thin ridges and branches

Closing

Fill small gaps and holes

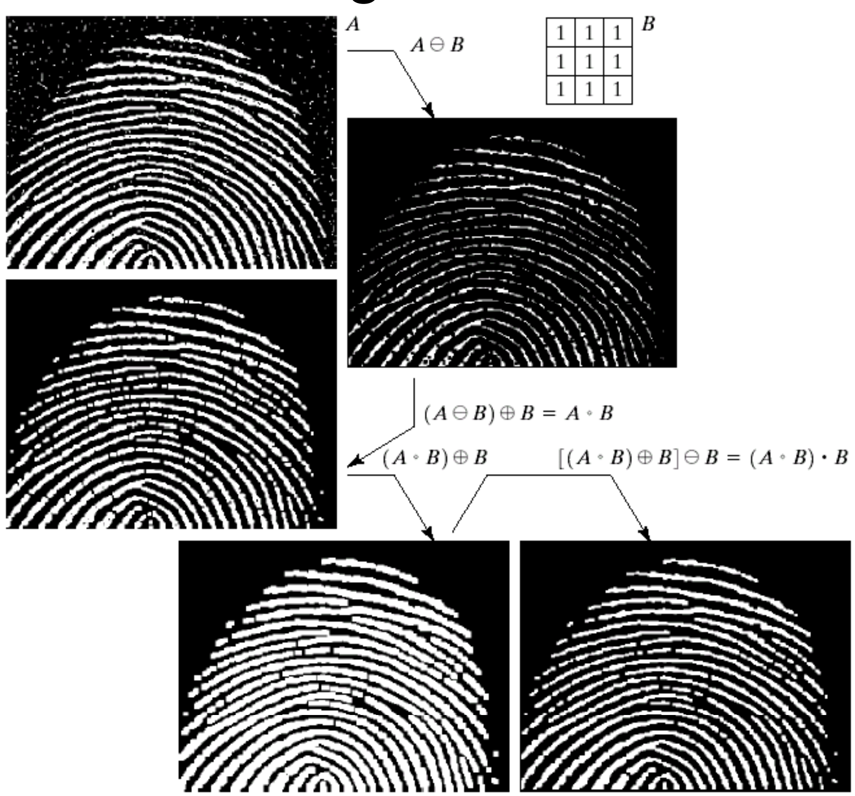
Example of Grayscale Opening and Closing

a b

FIGURE 9.31 (a) Opening and (b) closing of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Morphological Operation for Image Enhancement

- Morphological smoothing
  - Opening followed by closing, $(f \circ h) \bullet h$
  - Attenuated both bright and dark details

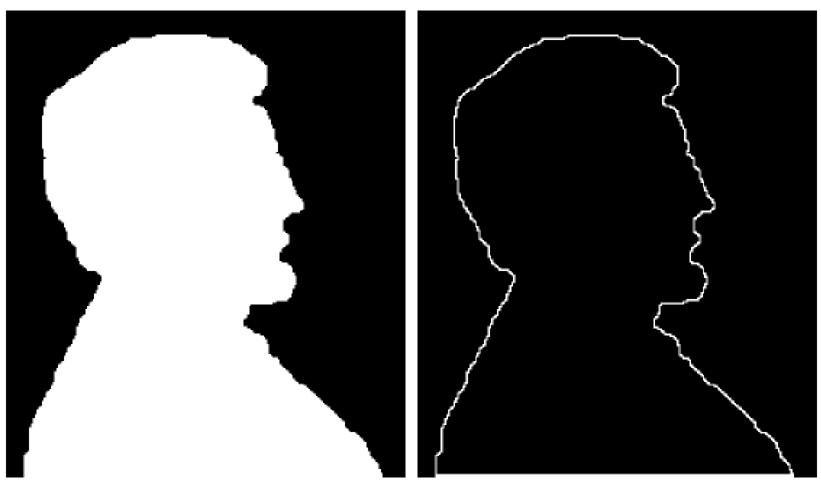


a b c d e f

FIGURE 9.11 (a) Noisy image. (c) Eroded image. (d) Opening of A. (d) Dilation of the opening. (e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

Morphological Operation for Image Enhancement

- Morphological gradient $(f \oplus h) - (f \Theta h)$
  - The difference between the dilated and eroded images,
- Valley detection $f \bullet h - f$
  - Detect dark text/lines from a gray background
- Boundary detection $f - f \Theta h$



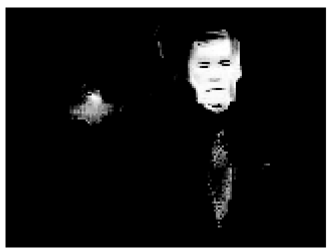
a b

FIGURE 9.14 (a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

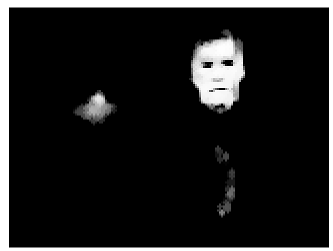
Application in Face Detection

- Use color information to detect candidate face region
- Verify the existence of face in candidate region

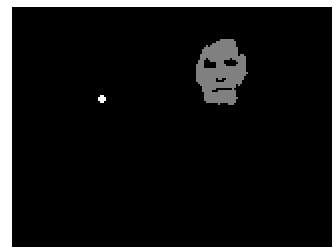
Input image



Skin-tone color likelihood



Opening processed image



Blob coloring

Face detection result

Written Assignment

1. For the image A in Figure 1(b), using the structuring element B in Figure 1(a), determine the closing and opening of A by B.

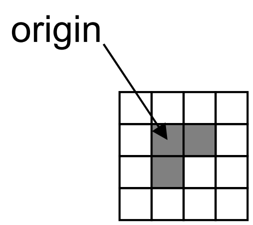


Figure 1. (a)

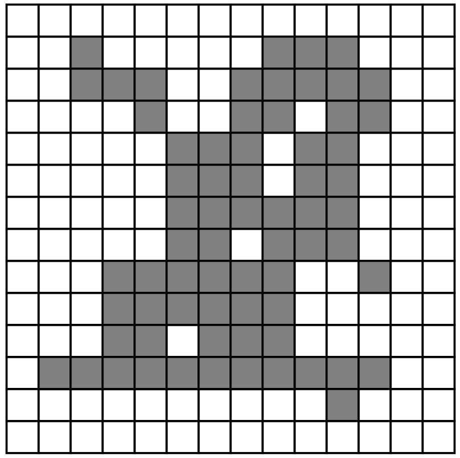


Figure 1. (b)

2. Consider a contiguous image function f and a gray scale structuring element h described by

$$f(x,y)=\begin{cases}1 & -4\le x,y\le 4;\\ 0 & otherwise\end{cases} \qquad h(x,y)=\begin{cases}1 & -1\le x,y\le 1;\\ 0 & otherwise\end{cases}$$

Derive the gray scale dilation and erosion of f by h, respectively.

Computer Assignment

1. Write a program which can i) add salt-and-pepper noise to an image with a specified noise density, ii) perform median filtering with a specified window size. Consider only median filter with a square shape. Try two different noise density (0.05, 0.2) and for each density, comment on the effect of median filtering with different window sizes and experimentally determine the best window size. You can use imnoise( ) to generate noise. You should write your own function for median filtering. You can ignore the boundary problem by only performing the filtering for the pixels inside the boundary.
2. In a previous assignment you have created a program for adding Gaussian noise and filtering using average filter. Apply the averaging filter and the median filter both to an image with Gaussian noise (with a chosen noise variance) and with salt-and-pepper noise (with a chosen noise density). Comment on the effectiveness of each filter on each type of noise.
3. Use the MATLAB program imdilate( ) and imerode( ) on a sample BW image. Try a simple 3x3 square structuring element. Comment on the effect of dilation and erosion. By concatenating the dilation and erosion operations, also generate result of closing and opening, and comment on their effects. Repeat above with a 7x7 structure element. Comment on the effect of the window size. You can generate a binary image from a grayscale one by thresholding.
4. Repeat above on a gray scale image, using MATLAB gray scale dilation and erosion functions.
5. Optional (for your own practice, will not be graded): write your own program for gray-scale dilation and erosion.

Reading

- R. Gonzalez, “Digital Image Processing,” Chapter 5.3 and Chapter 9.
- A. K. Jain, “Fundamentals of Digital Image Processing,” Section 7.4 and Section 9.9.