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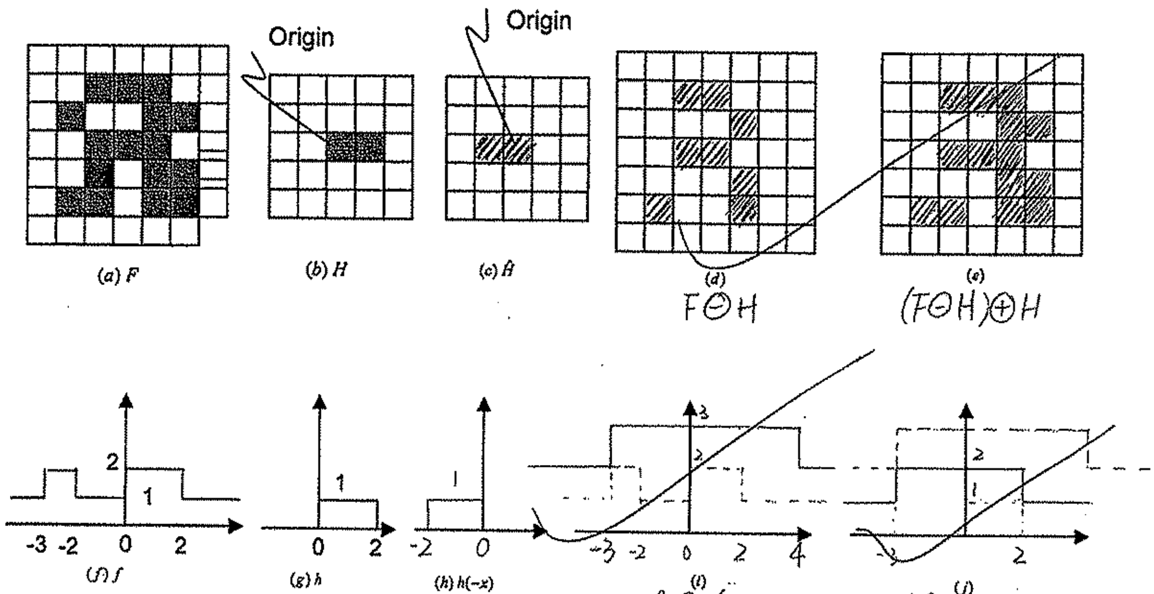
1. (20pt) (a) (10pt) Find the opening of the binary image, F, in Fig. (a) by the structuring element H in Fig. (b). You can use the grids in Fic. (c) to Fig. (e) to draw the intermediate and the final results.
(b) (10 pt) Find the gray scale closing of the function sketched in Fig. (f) with the structural element in Fig. (g). Sketch the intermediate and final result in Figs. (h) to Fig. (j). Comment on the effect of these operations.



(a) F (b) H (c) $\hat{H}$ (d) (e)

(f) f (g) h (h) h(−x) (i) (j)

horizontal

a) Obviously, the thin ridges and branches are eliminated comparing with the original image F.

b) Learning from the figure above, the small gaps between (-2,0) interval is filled after the operation.

2.

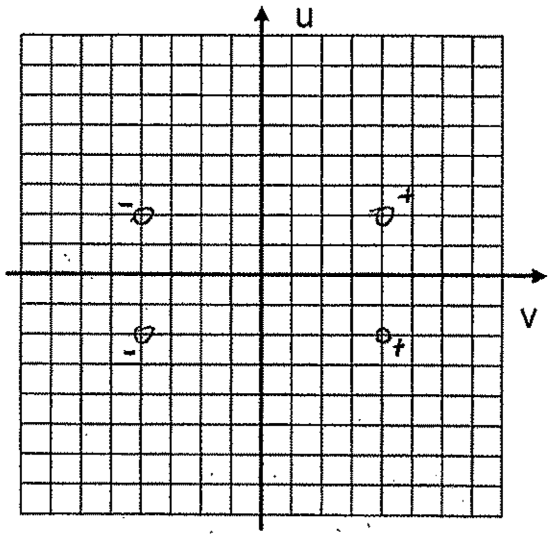


Fig.(a) Spectrum of original signal

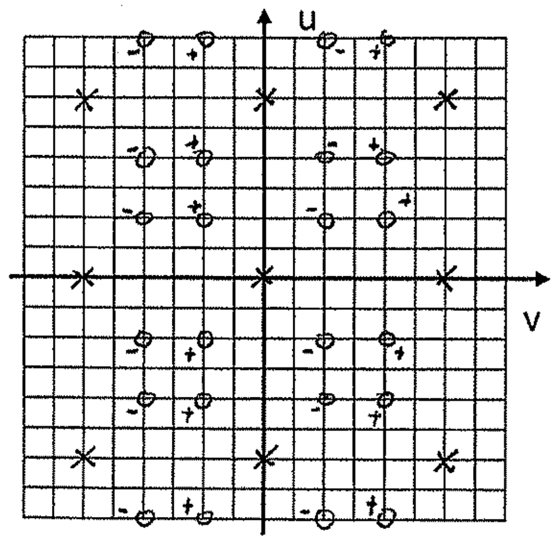


Fig.(b) Spectrum of sampled signal

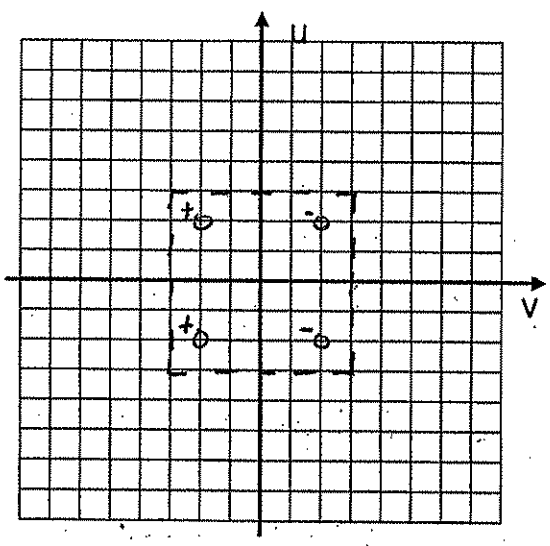


Fig.(c) Spectrum of interpolated signal by $H_1(u,v)$. Write down the spatial domain representation $f_{r1}(x,y)$ below

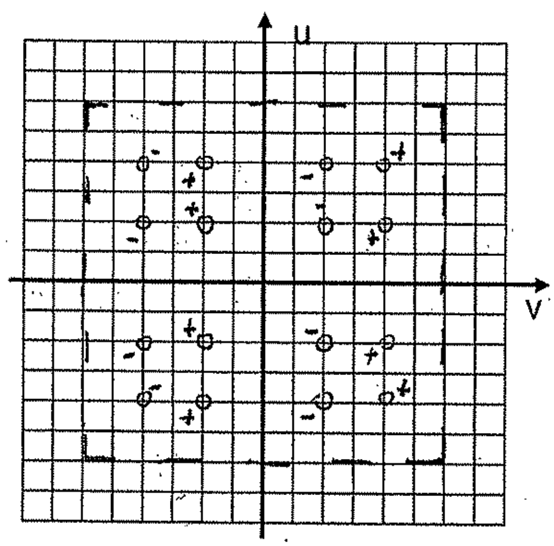


Fig.(d) Spectrum of interpolated signal by $H_2(u,v)$. Write down the spatial domain representation $f_{r2}(x,y)$ below.

2. a) $f(x,y) = \cos 4\pi x \sin 8\pi y = \frac{1}{2}[\sin(4\pi x + 8\pi y) - \sin(4\pi x - 8\pi y)]$

$\longleftrightarrow F(u,v) = \frac{1}{4j}[\delta(u-2,v-4) - \delta(u+2,v+4) - \delta(u-2,v+4) + \delta(u+2,v-4)]$

b) $f_s = 1/\Delta = 6$ Hz

c) $\frac{1}{2} f_s = 3$ Hz, with $-3<u,v<3$, there are 2 pairs of impulses, which are $(2,-2)$ and $(2,2)$

So, $F_{r_1} = \frac{1}{4j}[\delta(u-2,v+2) - \delta(u+2,v-2) - \delta(u-2,v-2) + \delta(u+2,v+2)]$

$f_{r_1} = \frac{1}{2}[\sin 2\pi(2x-2y) - \sin 2\pi(2x+2y)]$

$= -\cos 4\pi x \sin 4\pi y$

d) $H_x(u) = \begin{cases} -\frac{1}{f_s^2}|u| + \frac{1}{f_s}, & -f_s<u<f_s \\ 0, & \text{otherwise.} \end{cases}$

$H_2(u,v) = H_x(u)\cdot H_x(v) = \begin{cases} (\frac{1}{f_s} - \frac{1}{f_s^2}|u|)(\frac{1}{f_s} - \frac{1}{f_s^2}|v|), & -f_s<u,v<f_s \\ 0, & \text{otherwise.} \end{cases}$

within frequencies $-6<u,v<6$, there are 8 pairs of impulses, which are $(2,2), (2,-2), (4,4), (4,-4), (2,4), (2,-4), (4,2), (4,-2)$

$F_{r_2} = H_2(u,v)\cdot\frac{1}{\Delta^2}\cdot\frac{1}{4j}[\delta(u-2,v+2) - \delta(u+2,v-2) - \delta(u-2,v-2) + \delta(u+2,v+2)$
$- \delta(u-4,v+4) + \delta(u+4,v-4) + \delta(u-4,v-4) - \delta(u+4,v+4) + \delta(u-2,v-4) - \delta(u+2,v+4)$
$+ \delta(u+2,v-4) - \delta(u\overset{-2}{\cancel{-4}},v+4) + \delta(u-4,v-2) + \delta(u+4,v+2) + \delta(u-4,v+2) - \delta(u+4,v-2)]$

$f_{r_2} = \frac{1}{2\Delta^2}[H_2(2,-2)\sin 2\pi(2x-2y) - H_2(2,2)\sin 2\pi(2x+2y) - H_2(4,-4)\sin 2\pi(4x-4y)$
$+ H_2(4,4)\sin 2\pi(4x+4y) + H_2(2,4)\sin 2\pi(2x+4y) + H_2(-2,4)\sin 2\pi(-2x+4y)$
$- H_2(4,2)\sin 2\pi(4x+2y) + H_2(4,-2)\sin 2\pi(4x-2y)]$

$= -\frac{4}{9}\cos 4\pi x \sin 4\pi y + \frac{1}{9}\cos 8\pi x \sin 8\pi y + \frac{2}{9}\cos 4\pi x \sin 8\pi y - \frac{2}{9}\cos 8\pi x \sin 4\pi y$

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3) ⓐ The entropy is as follow:

$H = -\sum P_n \log_2 P_n = -(\frac{1}{2}\cdot\log_2\frac{1}{2} + \frac{3}{8}\log_2\frac{3}{8} + \frac{1}{8}\log_2 {}^1/_8) = 1.41$

the Huffman codebook:

$P(a)$ $1/2$ — 1 — 1

$P(b)$ $3/8$ — 1 — $\frac{1}{2}$ — 0 — 0 1

$P(c)$ $1/8$ — 0 — 0 0

$BPS = 1\times\frac{1}{2} + 2\times\frac{3}{8} + 2\times\frac{1}{8}$

$= 1.5$

which is larger than entropy

ⓑ Because the previous symbol is a,

$P(a|a)$ $2/3$ — 1 — 1

$P(b|a)$ $1/6$ — 1 — 0 — 0 1

$P(c|a)$ $1/6$ — 0 — 0 0

$H(Y|X=a) = -(\frac{2}{3}\log_2\frac{2}{3} + (\frac{1}{6}\log_2\frac{1}{6})\times 2) = 1.252$

$BPS = 2/3 + \frac{1}{6}\times 2\times 2 = 1.33$ which is bigger than entropy.

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4) (a) $h_{11} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $h_{12} = \frac{1}{2}\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$ $h_{21} = \frac{1}{2}\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$ $h_{22} = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

(b) $t_1 = 8$ $t_2 = -1$ $t_3 = -3$ $t_4 = 0$

(c) By using the function: $t_q = \lfloor (t + Q/2)/Q \rfloor * Q * \text{sgn}(t)$

$t_1' = 8$ $t_2' = -2$ $t_3' = -4$ $t_4' = 0$

(d) $F_{rec} = t_1' * h_{11} + t_2' * h_{12} + t_3' * h_{13} + t_4' * h_{14}$

$= \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 2 & 2 \\ -2 & -2 \end{bmatrix}$

$= \begin{bmatrix} 7 & 5 \\ 3 & 1 \end{bmatrix}$

5.a) $\langle h_1, h_1\rangle = \cos^2\theta + \sin^2\theta = 1$, $\langle h_1, h_2\rangle = -\sin\theta\cos\theta + \sin\theta\cos\theta = 0$

$\langle h_2, h_1\rangle = -\sin\theta\cos\theta + \cos\theta\sin\theta = 0$, $\langle h_2, h_2\rangle = (-\sin\theta)^2 + \cos^2\theta = 1$

$$\therefore \langle h_i, h_j\rangle = \begin{cases} 1 & i=j \ , \ i,j=1,2 \\ 0 & i\neq j \ , \ i,j=1,2 \end{cases}$$

$\therefore$ They form orthonormal basis for ~~any~~ arbitrary θ

b). $t = Af$, $\therefore C_t = AC_fA^T$, $A = \begin{bmatrix} h_1^H \\ h_2^H \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$$\therefore C_t = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta + \rho\sin\theta & \rho\cos\theta + \sin\theta \\ -\sin\theta + \rho\cos\theta & -\rho\sin\theta + \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \rho\sin\theta\cos\theta + \rho\sin\theta\cos\theta + \sin^2\theta & -\sin\theta\cos\theta - \rho\sin^2\theta + \rho\cos^2\theta + \sin\theta\cos\theta \\ -\sin\theta\cos\theta + \rho\cos^2\theta - \rho\sin^2\theta + \cos\theta\sin\theta & \sin^2\theta - \rho\sin\theta\cos\theta - \rho\sin\theta\cos\theta + \cos^2\theta \end{bmatrix}$$

$$= ! \begin{bmatrix} 1 + \rho\sin 2\theta & \rho\cos 2\theta \\ \rho\cos 2\theta & 1 - \rho\sin 2\theta \end{bmatrix}$$

c) Let $\cos 2\theta = 0 \Rightarrow 2\theta = \frac{\pi}{2}$, $\theta = \frac{\pi}{4}$

$$\therefore C_t = \begin{bmatrix} 1+\rho & 0 \\ 0 & 1-\rho \end{bmatrix}$$

6) a) $X = R(u+t) \longrightarrow$ forward

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$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

forward mapping

$$t = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\text{So: } \begin{bmatrix} X \\ y \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} u+5 \\ v+2 \end{bmatrix} = \begin{bmatrix} 1 + \frac{5\sqrt{3}}{2} + \frac{\sqrt{3}}{2}u + \frac{1}{2}v \\ \sqrt{3} - \frac{5}{2} - \frac{1}{2}u + \frac{\sqrt{3}}{2}v \end{bmatrix}$$

$$u = R^{-1}X - t$$

inverse mapping

$$= \begin{bmatrix} \sqrt{3}/2 & -\frac{1}{2} \\ \frac{1}{2} & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2}x - \frac{1}{2}y - 5 \\ \frac{1}{2}x + \frac{\sqrt{3}}{2}y - 2 \end{bmatrix} = \begin{bmatrix} u \\ v \end{bmatrix}$$

b) the mapping function is

$$\begin{cases} X = 1 + 5\sqrt{3}/2 + \frac{\sqrt{3}}{2}u + \frac{1}{2}v \\ y = \sqrt{3} - 5/2 - \frac{1}{2}u + \frac{\sqrt{3}}{2}v. \end{cases}$$

for f1 generating f2.

this is a affine mapping function

We should use inverse mapping to determine (u,v) for every pixel (x,y) in f2. Interpolate pixels in f1 to get its value at (u,v).

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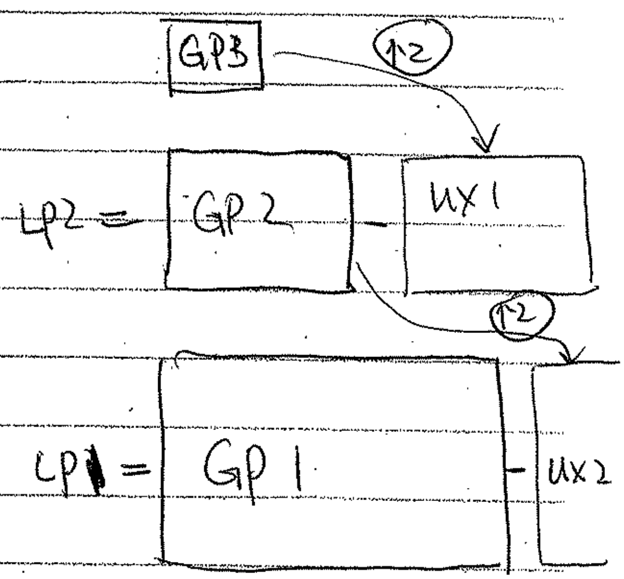
7. a) function [GP1, GP2, GP3] = GaussianPyramid(inimg)

```
inimg = imread('inimg.jpg');
[height, width] = size(inimg); GP1 = inimg;
hheight = height/2; hwidth = width/2; hhheight = height/4; hhwidth = width/4;
GP2 = zeros(hheight, hwidth);
for i = 1 : hheight
    for j = 1 : hwidth
        GP2(i,j) = mean(mean(GP1(i*2-1 : i*2+1, j*2-1, j*2+1))); % 3x3 averaging
    end
end
GP3 = zeros(hhheight, hhwidth);
  for i = 1 : hhheight
    for j = 1 : hhwidth
      GP3(i,j) = mean(mean(GP2(i*2-1 : i*2+1, j*2-1, j*2+1)));
      end
  end
figure;
subplot(1,3,1); imshow(GP1);
subplot(1,3,2); imshow(GP2);
subplot(1,3,3); imshow(GP3);
```

```
b) function [LP1, LP2, LP3] = LaplacianPyramid (GP1, GP2, GP3)
      imread ('GP3.jpg'); imread ('GP1.jpg'); imread ('GP2.jpg');
 [height, width] = size (GP1);
  hheight = height/2; hwidth = width/2;
   ux1 = zeros [hheight, hwidth];
     for i=1: hheight,
       for j=1: hwidth
          ux1(2*i-1, 2*j-1) = GP3(i, j);
          ux1(2*i-1, 2*j) = (GP3(i,j) + GP3(i, j+1)) * 0.5;
          ux1(2*i, 2*j-1) = (GP3(i,j) + GP3(i+1, j)) * 0.5;
          ux1(2*i, 2*j) = (GP3(i,j)+GP3(i,j+1)+GP3(i+1,j)+GP3(i+1,j+1))*0.25
          end
     end
    LP2 = zeros [hheight, hwidth];
     for i=1: hheight
       for j= 1: hwidth
          LP2(i,j) = GP2(i,j) - ux1(i,j);
        end
     end
```

LP2 = GP2 - ux1;




```
    ux2 = zeros [height, width]
     for i=1: height
      for j= 1: width
         ux2(2*i-1, 2*j-1) = GP2(i, j);
         ux2(2*i-1, 2*j) = (GP2(i,j) + GP2(i,j+1))*0.5;
         ux2(2*i, 2*j-1) = (GP2(i,j) + GP2(i+1,j)) * 0.5;
         ux2(2*i, 2*j) = (GP2(i,j) + GP2(i,j+1) + GP2(i+1,j) + GP2(i+1,j+1))*0.25
       end
     end
```

```
LP1 = zeros[height, width];
    for i = 1:height
      for j = 1:width
        LP1(i,j) = GP1(i,j) - UX2(i,j);
      end
    end
```

LP1 = GP1 - UX2

figure;

~~subplot(1,3,1); imshow(LP1);~~

subplot(1,3,2); imshow(LP2);

subplot(1,3,3); imshow(LP3);