A NOTE ON CIRCULAR CONVOLUTION

Suppose $x(n)$ and $g(n)$ are finite length signals with the same length:

$$x(n), \quad n = 0, \ldots, N-1$$

$$g(n), \quad n = 0, \ldots, N-1.$$

Conventional (linear) convolution is given by

$$\begin{aligned}(x * g)(n) &= \sum_k x(k) g(n-k) \\ &= x(0)\, g(n) + x(1)\, g(n-1) + \cdots + x(N-1)\, g(n-(N-1))\end{aligned}$$

So the linear convolution

$$y = [x(0) \;\; x(1) \;\; x(2) \;\; x(3)] * [g(0) \;\; g(1) \;\; g(2) \;\; g(3)]$$

can be written out as

$$\begin{aligned} y = x(0)\; & [g(0) \;\; g(1) \;\; g(2) \;\; g(3) \;\; 0 \;\; 0 \;\; 0] \\ + x(1)\; & [0 \;\; g(0) \;\; g(1) \;\; g(2) \;\; g(3) \;\; 0 \;\; 0] \\ + x(2)\; & [0 \;\; 0 \;\; g(0) \;\; g(1) \;\; g(2) \;\; g(3) \;\; 0] \\ + x(3)\; & [0 \;\; 0 \;\; 0 \;\; g(0) \;\; g(1) \;\; g(2) \;\; g(3)]. \end{aligned}$$

The sequence y is of length 7.

In circular convolution (or 'periodic convolution'), the shift $g(n-k)$ is taken to be a *circular* shift. So the circular convolution

$$y = [x(0) \;\; x(1) \;\; x(2) \;\; x(3)] \circledast [g(0) \;\; g(1) \;\; g(2) \;\; g(3)]$$

can be written out as

$$\begin{aligned} y = x(0)\; & [g(0) \;\; g(1) \;\; g(2) \;\; g(3)] \\ + x(1)\; & [g(3) \;\; g(0) \;\; g(1) \;\; g(2)] \\ + x(2)\; & [g(2) \;\; g(3) \;\; g(0) \;\; g(1)] \\ + x(3)\; & [g(1) \;\; g(2) \;\; g(3) \;\; g(0)] \end{aligned}$$

The sequence y is of length 4.