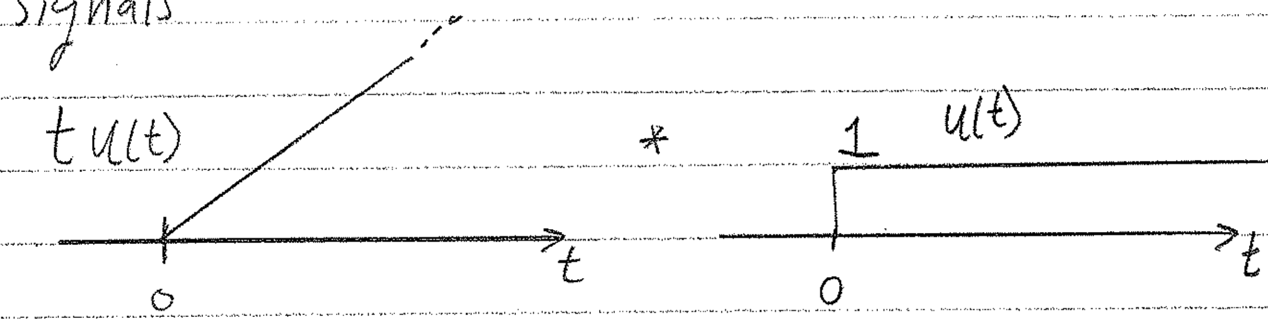
EE 3054 - TEST 2 - FALL 2008

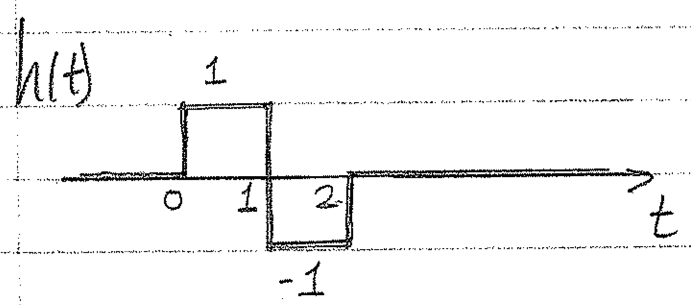
① $f(t) = u(t)$, $g(t) = 3e^{-2t}u(t)$

Find $x(t) = f(t) * g(t)$. (convolution.)
Also sketch $x(t)$.

② Find the convolution of the following two signals

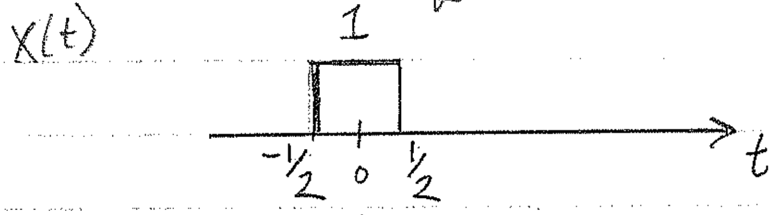


③ An LTI system has the impulse response shown:



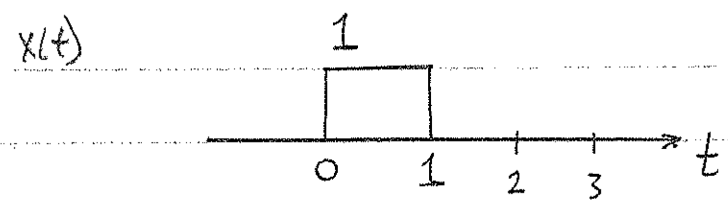
ⓐ Accurately sketch the step response of the system.
ⓑ Find the transfer function $H(s)$.

4. (a) Find the Fourier transform $X^f(\omega)$ of $x(t)$,
- write $X^f(\omega)$ using the sinc function.



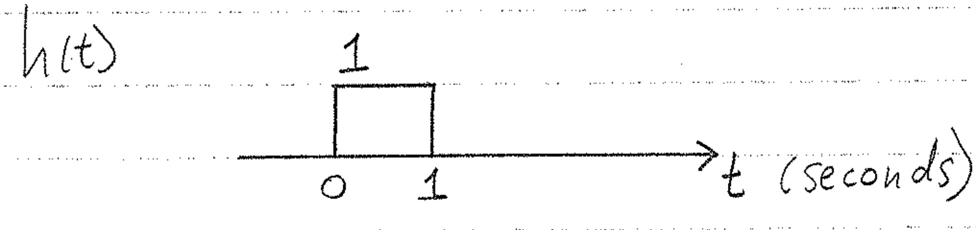
(b) sketch $X^f(\omega)$, indicate the nulls of $X^f(\omega)$.

5. (a) Find $X^f(\omega)$ for $x(t)$,



(b) sketch $|X^f(\omega)|$, indicate the nulls of $|X^f(\omega)|$, indicate the nulls of $|X^f(\omega)|$.

6. If an LTI system has the impulse response



then what frequencies are completely stopped by the system? Express the frequencies in Herz.

hint: use (#5)

⑦ An causal LTI system is described by the diff. eq.

$$y''(t) + 5y'(t) + 4y(t) = 2x'(t) + x(t).$$

a) Find the impulse response, $h(t)$.
b) Sketch the pole/zero diagram.

⑧ The impulse response of an LTI system is

$$h(t) = 2u(t) + 3e^{-2t}u(t)$$

a) Find the differential equation for the system.
b) sketch the pole/zero diagram.
c) classify the system as stable/unstable.

⑨ $x(t) = 2\cos(\pi t) + \cos(1.5\pi t)$

a) Find and sketch the Fourier Transform $X^f(\omega)$.

b) Find the Fourier Series coefficients $c(k)$ and the fundamental frequency ω_0,
c) Also, sketch the line spectrum of $x(t)$.

⑩ Find and sketch the Fourier transform of
$x(t) = \cos(3\pi t) \cdot \cos(4\pi t)$.

MATLAB PART

M 1) An EE 3054 student uses the following MATLAB code to implement a discrete-time system.

```
b=[4 2 1];
a=[3 5];
y=filter(b,a,x);
```

a) what is the transfer function $H(z)$ of the system?
b) " " " difference equation " " " ?

M 2) What is the result of the commands:

a) roots([1 1 -6])
b) poly([2 3 1])

M 3) Write MATLAB commands to produce a 10 second cosine signal of frequency 3.5 Hz, the sampling interval for your MATLAB command should be 0.1 seconds.

useful formulas

name	formula
Euler's formula	$e^{j\theta} = \cos(\theta) + j\sin(\theta)$
... for cosine	$\cos(\theta) = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$
... for sine	$\sin(\theta) = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$
sinc function	$\mathrm{sinc}(\theta) := \dfrac{\sin(\pi\,\theta)}{\pi\,\theta}$

formulas for continuous-time LTI signals and systems

name	formula
area under impulse	$\int \delta(t)\,dt = 1$
multiplication by impulse	$f(t)\,\delta(t) = f(0)\,\delta(t)$
... by shifted impulse	$f(t)\,\delta(t-t_o) = f(t_o)\,\delta(t-t_o)$
convolution	$f(t) * g(t) = \int f(\tau)\,g(t-\tau)\,d\tau$
... with an impulse	$f(t) * \delta(t) = f(t)$
... with a shifted impulse	$f(t) * \delta(t-t_o) = f(t-t_o)$
transfer function	$H(s) = \int h(t)\,e^{-st}\,dt$
frequency response	$H^f(\omega) = \int h(t)\,e^{-j\omega t}\,dt$
... their connection	$H^f(\omega) = H(j\omega)$ provided $j\omega$-axis $\subset$ ROC

selected Laplace transform pairs

$x(t)$	$X(s)$	ROC
$x(t)$	$\int x(t)\,e^{-st}\,dt$ (def.)	
$\delta(t)$	1	all s
$u(t)$	$\dfrac{1}{s}$	$\mathrm{Re}(s) > 0$
$e^{-a\,t}\,u(t)$	$\dfrac{1}{s+a}$	$\mathrm{Re}(s) > -a$
$\cos(\omega_o t)\,u(t)$	$\dfrac{s}{s^2+\omega_o^2}$	$\mathrm{Re}(s) > 0$
$\sin(\omega_o t)\,u(t)$	$\dfrac{\omega_o}{s^2+\omega_o^2}$	$\mathrm{Re}(s) > 0$
$e^{-a\,t}\cos(\omega_o t)\,u(t)$	$\dfrac{s+a}{(s+a)^2+\omega_o^2}$	$\mathrm{Re}(s) > -a$
$e^{-a\,t}\sin(\omega_o t)\,u(t)$	$\dfrac{\omega_o}{(s+a)^2+\omega_o^2}$	$\mathrm{Re}(s) > -a$

Note: a is assumed real.

Laplace transform properties

$x(t)$	$X(s)$
$a\,x(t) + b\,g(t)$	$a\,X(s) + b\,G(s)$
$x(t) * g(t)$	$X(s)\,G(s)$
$\dfrac{dx(t)}{dt}$	$s\,X(s)$
$x(t-t_o)$	$e^{-s\,t_o}\,X(s)$

selected Fourier transform pairs

$x(t)$	$X^f(\omega)$
$x(t)$	$\int x(t)\, e^{-j\omega t}\, dt$ (def.)
$\frac{1}{2\pi} \int X^f(\omega)\, e^{j\omega t}\, d\omega$	$X^f(\omega)$
$\delta(t)$	1
1	$2\,\pi\,\delta(\omega)$
$u(t)$	$\pi\,\delta(\omega) + \frac{1}{j\omega}$
$e^{j\omega_o t}$	$2\,\pi\,\delta(\omega - \omega_o)$
$\cos(\omega_o\, t)$	$\pi\,\delta(\omega + \omega_o) + \pi\,\delta(\omega - \omega_o)$
$\sin(\omega_o\, t)$	$j\,\pi\,\delta(\omega + \omega_o) - j\,\pi\,\delta(\omega - \omega_o)$
$\frac{\omega_o}{\pi}\,\mathrm{sinc}\left(\frac{\omega_o}{\pi} t\right)$	ideal LPF cut-off frequency ω_o
symmetric pulse width T, height 1	$\frac{2}{\omega}\,\sin\left(\frac{T}{2}\,\omega\right)$
impulse train period T, height 1	impulse train period, height $\omega_o = \frac{2\pi}{T}$

Fourier transform properties

$x(t)$	$X^f(\omega)$
$a\,x(t) + b\,g(t)$	$a\,X^f(\omega) + b\,G^f(\omega)$
$x(a\,t)$	$\frac{1}{\lvert a\rvert}\,X\left(\frac{\omega}{a}\right)$
$x(t) * g(t)$	$X^f(\omega)\,G^f(\omega)$
$x(t)\,g(t)$	$\frac{1}{2\pi}\,X^f(\omega) * G^f(\omega)$
$x(t - t_o)$	$e^{-jt_o\omega}\,X(\omega)$
$x(t)\,e^{j\omega_o t}$	$X(\omega - \omega_o)$
$x(t)\,\cos(\omega_o t)$	$0.5\,X(\omega + \omega_o) + 0.5\,X(\omega - \omega_o)$
$x(t)\,\sin(\omega_o t)$	$j\,0.5\,X(\omega + \omega_o) - j\,0.5\,X(\omega - \omega_o)$
$\frac{dx(t)}{dt}$	$j\,\omega\,X^f(\omega)$