EE 3054, QUIZ 2, FALL 2008.

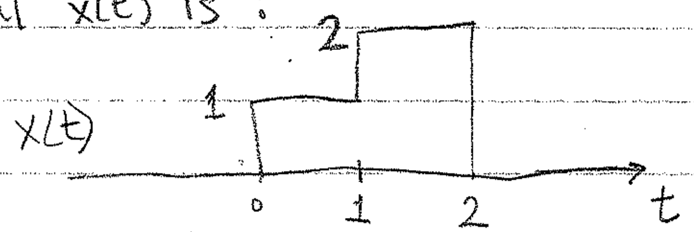
① $f(t) = \sum_{k=-\infty}^{\infty} \delta(t-3k)$ - sketch the signal $f(t)$.

② $g(t) = \dfrac{1}{1+t^2}$ - sketch $y(t) = g(t) \cdot f(t)$
where $f(t)$ is in ①.
$y(t)$ is the product of $g(t)$ and $f(t)$.

③ A continuous-time system is desribed by the rule

$$x(t) \longrightarrow \boxed{SYS} \longrightarrow y(t) = \int_{-\infty}^{t} [x(\tau)]^2 \, d\tau$$

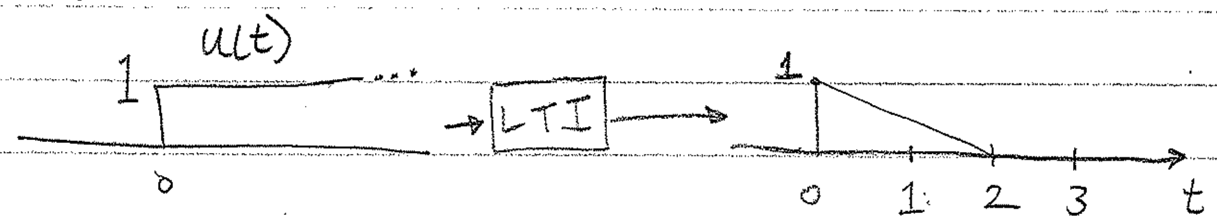
ⓐ sketch the output signal $y(t)$ when the input signal $x(t)$ is :



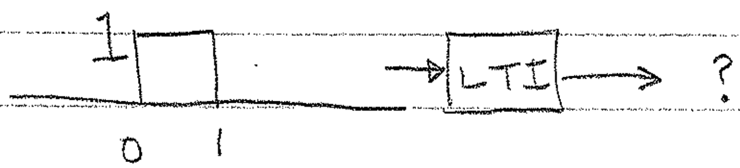
ⓑ is the system linear or non-linear?

ⓒ is " " time-invariant or time-varying?

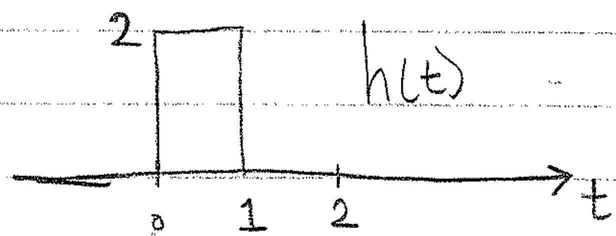
④ You observe of an unknown LTI system that



Accurately sketch the output signal produced by the input signal:

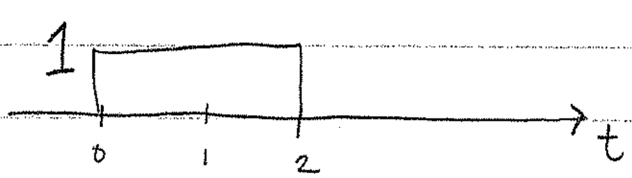


⑤ An LTI system has the impulse response



ⓐ What is the dc gain of the system?

ⓑ Accurately sketch the output signal produced by the input signal



ⓒ What is the transfer function $H(s)$?

⑥ Accurately sketch the convolution of the two signals:

$$f(t) = \delta(t) + \tfrac{1}{3}\,\delta(t-0.5)$$
$$g(t) = \delta(t-1) + u(t-2)$$

⑦ The impulse response of a C-T LTI system is

$$h(t) = 3\,\delta(t) + 2\,e^{-t/4}\cos(2\pi t)\,u(t)$$

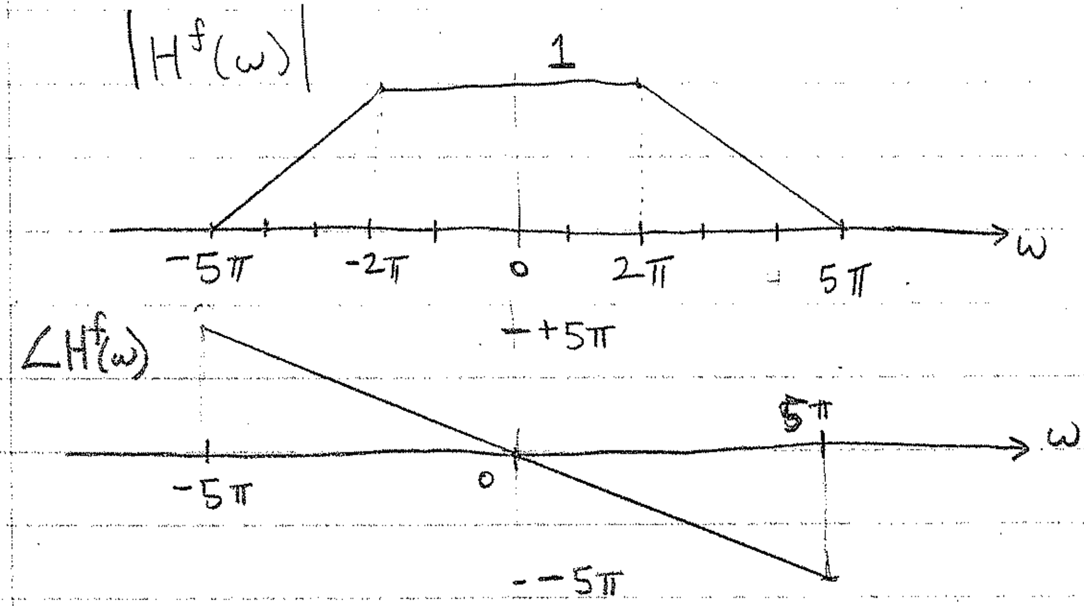
ⓐ list the poles of the system.
ⓑ find the differential equation for the system.
ⓒ what is the dc gain of the system?
ⓓ sketch the pole/zero diagram of the system.

⑧ $y''(t) + 5y'(t) + 7y(t) = A\,x''(t) + B\,x'(t) + C\,x(t)$
describes an LTI system.
Find constants A, B, C so that the system
ⓐ kills tones with frequencies 10 Hz, and
ⓑ has unity dc-gain.

⑨ An causal LTI system diff eq is : $y'(t) + 2y(t) = 3x(t)$.
Find the output $y(t)$ produced by the input signal

$$x(t) = 4e^{-t}u(t).$$

⑩ The frequency response of an LTI system is



Find the output signal produced by the input signal

$$x(t) = 3 + \sin(\pi t) + \cos(4\pi t) + \sin(5\pi t).$$

$$x(t) \longrightarrow \boxed{H} \longrightarrow y(t) = ?$$

⑪ The freq. response of an LTI system is

$$H^f(\omega) = \begin{cases} 0 & |\omega| < 2\pi \\ j\omega & 2\pi \le |\omega| \le 4\pi \\ 0 & |\omega| \geq 4\pi \end{cases}$$

ⓐ Sketch $|H^f(\omega)|$ and $\angle H^f(\omega)$.

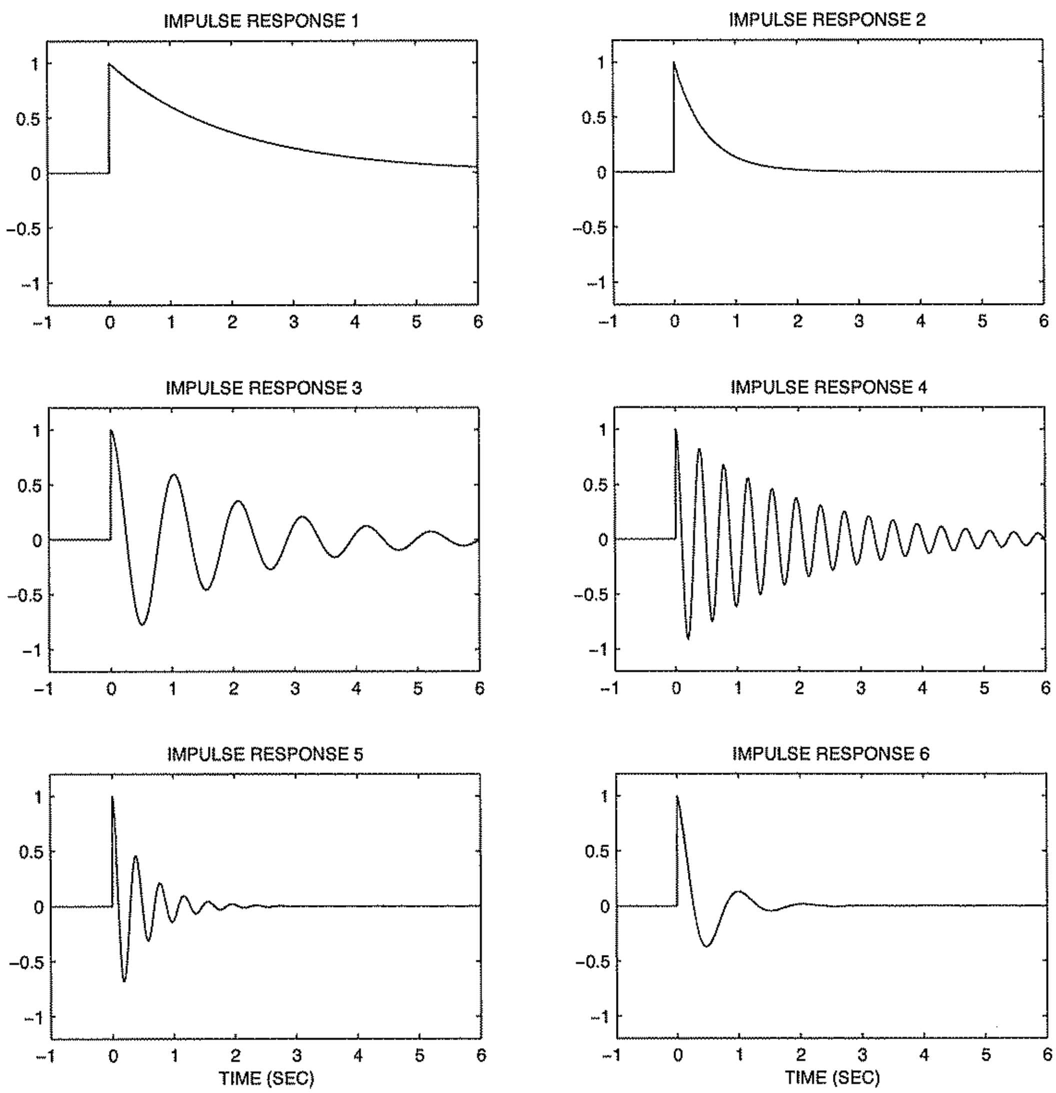
ⓑ Find the output signal $y(t)$ produced by the input signal

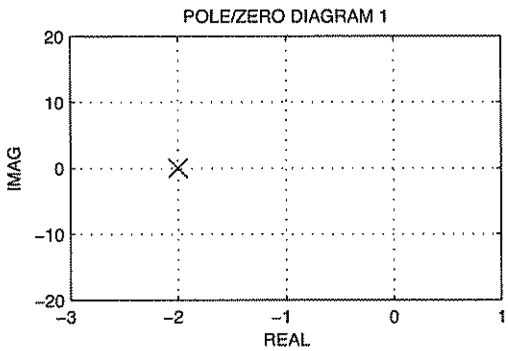
$$x(t) = 3 + 2\sin(\pi t) + 3\cos(3\pi t) + \cos(5\pi t).$$

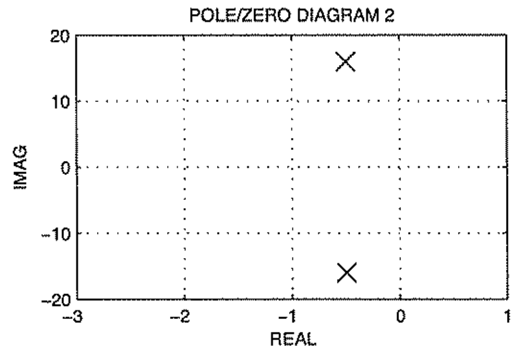
12.)

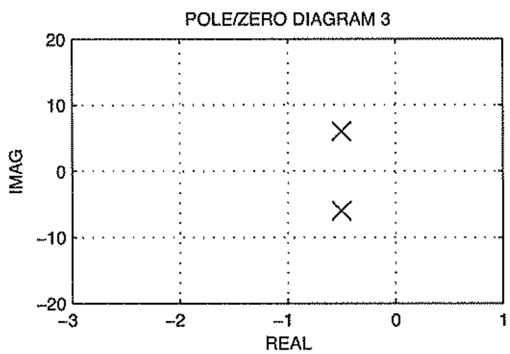
The impulse response, frequency response, and pole-zero diagram of 6 continuous-time LTI systems are illustrated below. However, they are out of order. Match each frequency response and pole-zero diagram to each impulse response by filling out the table. (Copy the table into your exam book.)

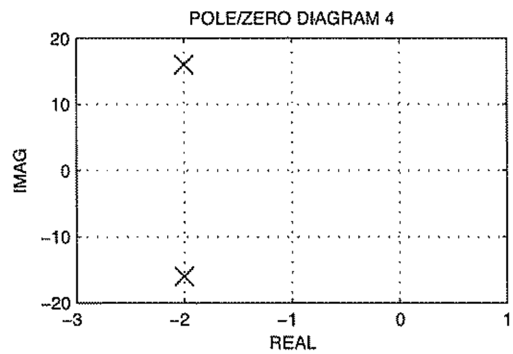
Impulse Response	Pole-Zero Diagram	Frequency Response
1		
2		
3		
4		
5		
6		

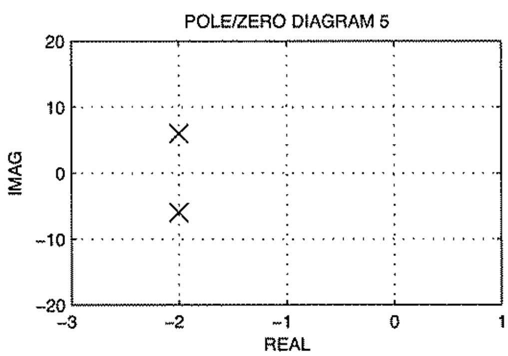


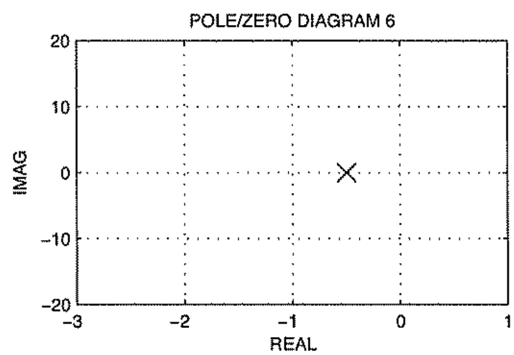

POLE/ZERO DIAGRAM 1
IMAG
20
10
0
−10
−20
−3
−2
−1
0
1
REAL

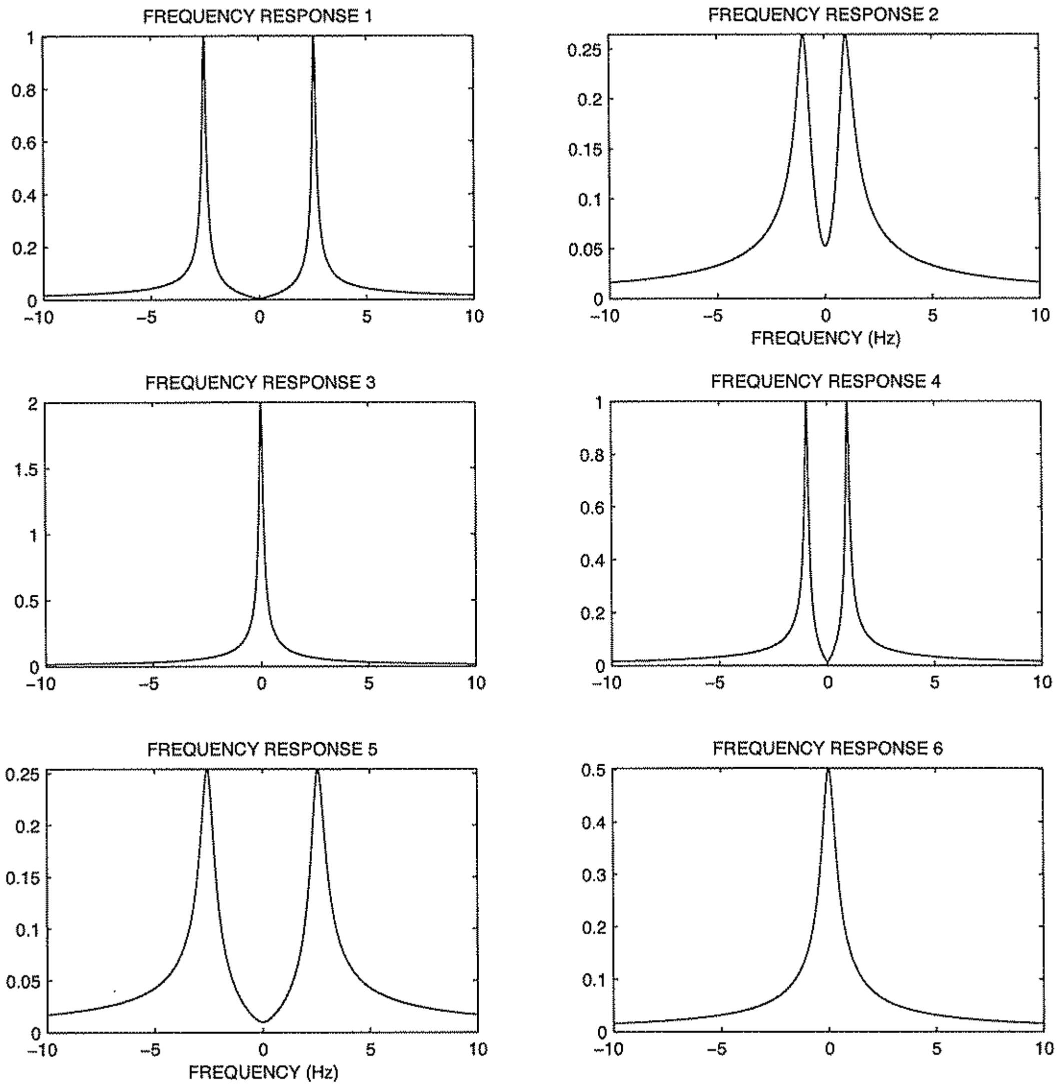

POLE/ZERO DIAGRAM 2
IMAG
20
10
0
−10
−20
−3
−2
−1
0
1
REAL


POLE/ZERO DIAGRAM 3
IMAG
20
10
0
−10
−20
−3
−2
−1
0
1
REAL


POLE/ZERO DIAGRAM 4
IMAG
20
10
0
−10
−20
−3
−2
−1
0
1
REAL


POLE/ZERO DIAGRAM 5
IMAG
20
10
0
−10
−20
−3
−2
−1
0
1
REAL


POLE/ZERO DIAGRAM 6
IMAG
20
10
0
−10
−20
−3
−2
−1
0
1
REAL


FREQUENCY RESPONSE 1
FREQUENCY RESPONSE 2
FREQUENCY (Hz)
FREQUENCY RESPONSE 3
FREQUENCY RESPONSE 4
FREQUENCY RESPONSE 5
FREQUENCY (Hz)
FREQUENCY RESPONSE 6

some formulas

name	formula
convolution	$f(t) * g(t) = \int f(\tau)\, g(t-\tau)\, d\tau$
transfer function	$H(s) = \int h(t)\, e^{-st}\, dt$
frequency response	$H^f(\omega) = \int h(t)\, e^{-j\omega t}\, dt$
... their connection	$H^f(\omega) = H(j\omega)$ provided $j\omega$-axis $\subset$ ROC
Euler's formula	$e^{j\theta} = \cos(\theta) + j\,\sin(\theta)$
... for cosine	$\cos(\theta) = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$
... for sine	$\sin(\theta) = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$

selected Laplace transform pairs

$x(t)$	$X(s)$	ROC
$x(t)$	$\int x(t)\, e^{-st}\, dt$ (def.)	
$\delta(t)$	1	all s
$u(t)$	$\dfrac{1}{s}$	$\mathrm{Re}(s) > 0$
$e^{-at}\, u(t)$	$\dfrac{1}{s+a}$	$\mathrm{Re}(s) > -a$
$\cos(\omega_o t)\, u(t)$	$\dfrac{s}{s^2+\omega_o^2}$	$\mathrm{Re}(s) > 0$
$\sin(\omega_o t)\, u(t)$	$\dfrac{\omega_o}{s^2+\omega_o^2}$	$\mathrm{Re}(s) > 0$
$e^{-at}\cos(\omega_o t)\, u(t)$	$\dfrac{s+a}{(s+a)^2+\omega_o^2}$	$\mathrm{Re}(s) > -a$
$e^{-at}\sin(\omega_o t)\, u(t)$	$\dfrac{\omega_o}{(s+a)^2+\omega_o^2}$	$\mathrm{Re}(s) > -a$

Note: a is assumed real.

Laplace transform properties

$x(t)$	$X(s)$
$a\,x(t) + b\,g(t)$	$a\,X(s) + b\,G(s)$
$x(t) * g(t)$	$X(s)\,G(s)$
$\dfrac{dx(t)}{dt}$	$s\,X(s)$
$x(t-t_o)$	$e^{-s\,t_o}\,X(s)$