**EE 105: Signals, Systems, and Transforms** **Spring 2002**

**Test 1**

- *No notes allowed; closed book.*
- *Only basic non-graphing non-programmable calculators are allowed.*
- *Simplify your answers. For partial credit please show your work.*
- *Please put a box around your* $\boxed{\textit{answers.}}$

---

The convolution of two signals $x(n)$ and $h(n)$ is given by

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k).$$

The $Z$-transform of

$$x(n) = a^n\, u(n) \qquad \text{is} \qquad X(z) = \frac{z}{z-a}, \qquad |z| > |a|.$$

The $Z$-transform of

$$x(n) = -a^n\, u(-n-1) \qquad \text{is} \qquad X(z) = \frac{z}{z-a}, \qquad |z| < |a|.$$

The signal $u(n)$ represents the discrete-time step function.
The signal $\delta(n)$ represents the discrete-time impulse function.

---

[5 %] 1. Carefully sketch the following signal

$$x(n) = \delta(n-2) \cdot \delta(n-4) + \delta(n+3)$$

---

[5 %] 2. Carefully sketch the following signal

$$x(n) = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^{|k|} \delta(n - 2\,k)$$

---

[5 %] 3. A discrete-time system is described by the following rule

$$y(n) = x(n^2)$$

Classify the system as:

(a) memoryless/with memory
(b) causal/noncausal
(c) linear/nonlinear
(d) time-invariant/time-varying
(e) BIBO stable/unstable

---

[5 %] 4. The $Z$-transform of a signal is given by

$$X(z) = z^3 + 1 - z^{-2}$$

(a) Sketch $x(n)$.
(b) Define $G(z) = z^{-2}X(1/z)$. Sketch $g(n)$.

---

[5 %] 5. Consider the two signals $g(n)$ and $f(n)$

$$f(n) = 2\,\delta(n) + 2\,\delta(n-1)$$

$$g(n) = \delta(n) + 2\,\delta(n-2) + 3\,\delta(n-4)$$

Sketch the convolution $f(n) * g(n)$. Show your work.

---

[10 %] 6. Consider the signal $g(n)$

$$g(n) = 2^n\, u(-n)$$

(a) Sketch $g(n)$.
(b) Find the $Z$-transform $G(z)$ of the signal and its ROC. Show your work.

---

[10 %] 7. Consider the two signals $g(n)$ and $f(n)$

$$f(n) = \left(\frac{1}{2}\right)^n u(n)$$

$$g(n) = 2^n\, u(-n)$$

Find and sketch the convolution $x(n) = f(n) * g(n)$.
Note, the ROC of $X(z)$ is the intersection of the ROC of $F(z)$ and $G(z)$.

---

[10 %] 8. The impulse response of a discrete-time LTI system is

$$h(n) = \delta(n-3).$$

(a) What is the transfer function $H(z)$ of the system?

(b) What is the frequency response $H^f(\omega)$ of the system?

(c) What is the frequency response magnitude $|H^f(\omega)|$?

(d) Find the output $y(n)$ when the following input signal is applied to the system.

$$x(n) = \cos(0.3\,\pi\,n)$$

(e) Find the step response of the system.

(f) Find the difference equation to describe the system.

(g) Classify the system as BIBO stable/unstable.

---

[10 %] 9. The transfer function of a causal discrete-time LTI system is

$$H(z) = \frac{1 + 2\,z^{-1} + 3\,z^{-2}}{1 - 3\,z^{-1}}.$$

(a) Find the difference equation to describe this system.

(b) List the poles of this transfer function.

(c) Classify the system as BIBO stable/unstable and explain your answer.

---

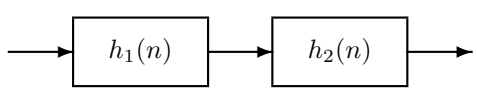
[10 %] 10. A causal discrete-time LTI system is described by the difference equation

$$y(n) = x(n) + y(n-2).$$

Find and sketch the impulse response.

---

[5 %] 11. Consider the cascade of two discrete-time LTI systems with impulse responses $h_1(n)$ and $h_2(n)$



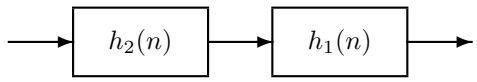

where

$$h_1(n) = 2\left(\frac{2}{3}\right)^n u(n).$$

and

$$h_{tot}(n) = 3\left(\frac{2}{3}\right)^n u(n) + 2\left(\frac{1}{3}\right)^n u(n).$$

Now suppose the order of the two systems are interchanged, so that the total system is as shown.




What is the impulse response of the total system? Make clear how you arrive at your answer.

---

[10 %] 12. Consider a discrete-time LTI system with the impulse response

$$h(n) = \delta(n) - \frac{7}{2}\,\delta(n-1) + \frac{3}{2}\,\delta(n-2).$$

Find a *bounded* signal $g(n)$ so that

$$h(n) * g(n) = \delta(n).$$

---

[10 %] 13. The frequency response $H^f(\omega)$ of a discrete-time LTI system is

$$H^f(\omega) = \begin{cases} e^{-j\,2.3\,\omega} & -0.4\pi < \omega < 0.4\pi \\ 0 & 0.4\pi < |\omega| < \pi. \end{cases}$$

Find the output $y(n)$ when the input $x(n)$ is

$$x(n) = 1.2\,\sin(0.3\,\pi\,n) + 1.5\,\sin(0.5\,\pi\,n).$$

Put $y(n)$ in simplest real form (your answer should not contain $j$).

Hint: Use Euler's formula and the relation

$$e^{j\omega_o n} \longrightarrow \boxed{\text{LTI SYSTEM}} \longrightarrow H^f(\omega_o)\,e^{j\omega_o n}$$

---

14. **Extra credit:** A causal discrete-time LTI system is described by the difference equation

$$y(n) = x(n) - y(n-2).$$

Find and sketch the impulse response. Simplify your answer so that it does not contain $j$.
Hint: $j$ can be written as $e^{j\,\frac{\pi}{2}}$, and $-j$ can be written as $e^{-j\,\frac{\pi}{2}}$.

---