

formulas for discrete-time LTI signals and systems

name	formula
area under impulse	$\sum_n \delta(n) = 1$
multiplication by impulse	$f(n) \delta(n) = f(0) \delta(n)$
... by shifted impulse	$f(n) \delta(n - n_o) = f(n_o) \delta(n - n_o)$
convolution	$f(n) * g(n) = \sum_k f(k) g(n - k)$
... with an impulse	$f(n) * \delta(n) = f(n)$
... with a shifted impulse	$f(n) * \delta(n - n_o) = f(n - n_o)$
transfer function	$H(z) = \sum_n h(n) z^{-n}$
frequency response	$H^f(\omega) = \sum_n h(n) e^{-j\omega n}$
... their connection	$H^f(\omega) = H(e^{j\omega})$ provided unit circle \subset ROC

formulas for continuous-time LTI signals and systems

name	formula
area under impulse	$\int \delta(t) dt = 1$
multiplication by impulse	$f(t) \delta(t) = f(0) \delta(t)$
... by shifted impulse	$f(t) \delta(t - t_o) = f(t_o) \delta(t - t_o)$
convolution	$f(t) * g(t) = \int f(\tau) g(t - \tau) d\tau$
... with an impulse	$f(t) * \delta(t) = f(t)$
... with a shifted impulse	$f(t) * \delta(t - t_o) = f(t - t_o)$
transfer function	$H(s) = \int h(t) e^{-st} dt$
frequency response	$H^f(\omega) = \int h(t) e^{-j\omega t} dt$
... their connection	$H^f(\omega) = H(j\omega)$ provided $j\omega$ -axis \subset ROC

useful formulas

name	formula
Euler's formula	$e^{j\theta} = \cos(\theta) + j \sin(\theta)$
... for cosine	$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$
... for sine	$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$
sinc function	$\text{sinc}(\theta) := \frac{\sin(\pi \theta)}{\pi \theta}$

Z-transform transform pairs

$x(n)$	$X(z)$	ROC
$x(n)$	$\sum_n x(n) z^{-n}$ (def.)	
$\delta(n)$	1	all z
$u(n)$	$\frac{z}{z - 1}$	$ z > 1$
$a^n u(n)$	$\frac{z}{z - a}$	$ z > a $
$-a^n u(-n - 1)$	$\frac{z}{z - a}$	$ z < a $
$\cos(\omega_o n) u(n)$	$\frac{z^2 - \cos(\omega_o) z}{z^2 - 2 \cos(\omega_o) z + 1}$	$ z > 1$
$\sin(\omega_o n) u(n)$	$\frac{\sin(\omega_o) z}{z^2 - 2 \cos(\omega_o) z + 1}$	$ z > 1$
$a^n \cos(\omega_o n) u(n)$	$\frac{z^2 - a \cos(\omega_o) z}{z^2 - 2 a \cos(\omega_o) z + a^2}$	$ z > a $
$a^n \sin(\omega_o n) u(n)$	$\frac{a \sin(\omega_o) z}{z^2 - 2 a \cos(\omega_o) z + a^2}$	$ z > a $

Z-transform transform properties

$x(n)$	$X(z)$
$a x(n) + b g(n)$	$a X(z) + b G(z)$
$x(n - n_o)$	$z^{-n_o} X(z)$
$x(n) * f(n)$	$X(z) F(z)$

selected Laplace transform pairs

$x(t)$	$X(s)$	ROC
$x(t)$	$\int x(t) e^{-st} dt$ (def.)	
$\delta(t)$	1	all s
$u(t)$	$\frac{1}{s}$	$\text{Re}(s) > 0$
$e^{-at} u(t)$	$\frac{1}{s+a}$	$\text{Re}(s) > -a$
$\cos(\omega_o t) u(t)$	$\frac{s}{s^2 + \omega_o^2}$	$\text{Re}(s) > 0$
$\sin(\omega_o t) u(t)$	$\frac{\omega_o}{s^2 + \omega_o^2}$	$\text{Re}(s) > 0$
$e^{-at} \cos(\omega_o t) u(t)$	$\frac{s+a}{(s+a)^2 + \omega_o^2}$	$\text{Re}(s) > -a$
$e^{-at} \sin(\omega_o t) u(t)$	$\frac{\omega_o}{(s+a)^2 + \omega_o^2}$	$\text{Re}(s) > -a$

Note: a is assumed real.

Laplace transform properties

$x(t)$	$X(s)$
$a x(t) + b g(t)$	$a X(s) + b G(s)$
$x(t) * g(t)$	$X(s) G(s)$
$\frac{dx(t)}{dt}$	$s X(s)$
$x(t - t_o)$	$e^{-s t_o} X(s)$

Fourier series

If $x(t)$ is periodic with period T then

$$x(t) = \sum c(k) e^{j k \omega_o t}$$

where

$$\omega_o = \frac{2\pi}{T}$$

and

$$c(k) = \frac{1}{T} \int_{\langle T \rangle} x(t) e^{-j k \omega_o t} dt$$

selected Fourier transform pairs

$x(t)$	$X^f(\omega)$
$x(t)$	$\int x(t) e^{-j\omega t} dt$ (def.)
$\frac{1}{2\pi} \int X^f(\omega) e^{j\omega t} d\omega$	$X^f(\omega)$
$\delta(t)$	1
1	$2\pi \delta(\omega)$
$u(t)$	$\pi \delta(\omega) + \frac{1}{j\omega}$
$e^{j\omega_o t}$	$2\pi \delta(\omega - \omega_o)$
$\cos(\omega_o t)$	$\pi \delta(\omega + \omega_o) + \pi \delta(\omega - \omega_o)$
$\sin(\omega_o t)$	$j\pi \delta(\omega + \omega_o) - j\pi \delta(\omega - \omega_o)$
$\frac{\omega_o}{\pi} \text{sinc}\left(\frac{\omega_o}{\pi} t\right)$	ideal LPF cut-off frequency ω_o
symmetric pulse width T , height 1	$\frac{2}{\omega} \sin\left(\frac{T}{2} \omega\right)$
impulse train period T , height 1	impulse train period, height $\omega_o = \frac{2\pi}{T}$

Fourier transform properties

$x(t)$	$X^f(\omega)$
$a x(t) + b g(t)$	$a X^f(\omega) + b G^f(\omega)$
$x(at)$	$\frac{1}{ a } X\left(\frac{\omega}{a}\right)$
$x(t) * g(t)$	$X^f(\omega) G^f(\omega)$
$x(t) g(t)$	$\frac{1}{2\pi} X^f(\omega) * G^f(\omega)$
$x(t - t_o)$	$e^{-j t_o \omega} X(\omega)$
$x(t) e^{j\omega_o t}$	$X(\omega - \omega_o)$
$x(t) \cos(\omega_o t)$	$0.5 X(\omega + \omega_o) + 0.5 X(\omega - \omega_o)$
$x(t) \sin(\omega_o t)$	$j 0.5 X(\omega + \omega_o) - j 0.5 X(\omega - \omega_o)$
$\frac{dx(t)}{dt}$	$j \omega X^f(\omega)$