formulas for discrete-time LTI signals and systems

name	formula
area under impulse	$\sum_n \delta(n) = 1$
multiplication by impulse	$f(n)\,\delta(n) = f(0)\,\delta(n)$
... by shifted impulse	$f(n)\,\delta(n-n_o) = f(n_o)\,\delta(n-n_o)$
convolution	$f(n) * g(n) = \sum_k f(k)\,g(n-k)$
... with an impulse	$f(n) * \delta(n) = f(n)$
... with a shifted impulse	$f(n) * \delta(n-n_o) = f(n-n_o)$
transfer function	$H(z) = \sum_n h(n)\,z^{-n}$
frequency response	$H^f(\omega) = \sum_n h(n)\,\mathrm{e}^{-\mathrm{j}\omega n}$
... their connection	$H^f(\omega) = H(\mathrm{e}^{\mathrm{j}\omega})$ provided unit circle $\subset$ ROC

formulas for continuous-time LTI signals and systems

name	formula
area under impulse	$\int \delta(t)\,dt = 1$
multiplication by impulse	$f(t)\,\delta(t) = f(0)\,\delta(t)$
... by shifted impulse	$f(t)\,\delta(t-t_o) = f(t_o)\,\delta(t-t_o)$
convolution	$f(t) * g(t) = \int f(\tau)\,g(t-\tau)\,d\tau$
... with an impulse	$f(t) * \delta(t) = f(t)$
... with a shifted impulse	$f(t) * \delta(t-t_o) = f(t-t_o)$
transfer function	$H(s) = \int h(t)\,\mathrm{e}^{-st}\,dt$
frequency response	$H^f(\omega) = \int h(t)\,\mathrm{e}^{-\mathrm{j}\omega t}\,dt$
... their connection	$H^f(\omega) = H(\mathrm{j}\omega)$ provided $\mathrm{j}\omega$-axis $\subset$ ROC

useful formulas

name	formula
Euler's formula	$\mathrm{e}^{\mathrm{j}\theta} = \cos(\theta) + \mathrm{j}\sin(\theta)$
... for cosine	$\cos(\theta) = \dfrac{\mathrm{e}^{\mathrm{j}\theta} + \mathrm{e}^{-\mathrm{j}\theta}}{2}$
... for sine	$\sin(\theta) = \dfrac{\mathrm{e}^{\mathrm{j}\theta} - \mathrm{e}^{-\mathrm{j}\theta}}{2\,\mathrm{j}}$
sinc function	$\mathrm{sinc}(\theta) := \dfrac{\sin(\pi\,\theta)}{\pi\,\theta}$

Z-transform transform pairs

$x(n)$	$X(z)$	ROC
$x(n)$	$\sum_n x(n)\,z^{-n}$ (def.)	
$\delta(n)$	1	all z
$u(n)$	$\dfrac{z}{z-1}$	$\|z\| > 1$
$a^n\,u(n)$	$\dfrac{z}{z-a}$	$\|z\| > \|a\|$
$-a^n\,u(-n-1)$	$\dfrac{z}{z-a}$	$\|z\| < \|a\|$
$\cos(\omega_o n)\,u(n)$	$\dfrac{z^2 - \cos(\omega_o)\,z}{z^2 - 2\,\cos(\omega_o)\,z + 1}$	$\|z\| > 1$
$\sin(\omega_o n)\,u(n)$	$\dfrac{\sin(\omega_o)\,z}{z^2 - 2\,\cos(\omega_o)\,z + 1}$	$\|z\| > 1$
$a^n\cos(\omega_o n)\,u(n)$	$\dfrac{z^2 - a\cos(\omega_o)\,z}{z^2 - 2\,a\cos(\omega_o)\,z + a^2}$	$\|z\| > \|a\|$
$a^n\sin(\omega_o n)\,u(n)$	$\dfrac{a\sin(\omega_o)\,z}{z^2 - 2\,a\cos(\omega_o)\,z + a^2}$	$\|z\| > \|a\|$

Z-transform transform properties

$x(n)$	$X(z)$
$a\,x(n) + b\,g(n)$	$a\,X(z) + b\,G(z)$
$x(n-n_o)$	$z^{-n_o}\,X(z)$
$x(n) * f(n)$	$X(z)\,F(z)$

selected Laplace transform pairs

$x(t)$	$X(s)$	ROC
$x(t)$	$\int x(t)\,\mathrm{e}^{-st}\,dt$ (def.)	
$\delta(t)$	1	all s
$u(t)$	$\frac{1}{s}$	$\mathrm{Re}(s) > 0$
$\mathrm{e}^{-a\,t}\,u(t)$	$\frac{1}{s+a}$	$\mathrm{Re}(s) > -a$
$\cos(\omega_o t)\,u(t)$	$\frac{s}{s^2+\omega_o^2}$	$\mathrm{Re}(s) > 0$
$\sin(\omega_o t)\,u(t)$	$\frac{\omega_o}{s^2+\omega_o^2}$	$\mathrm{Re}(s) > 0$
$\mathrm{e}^{-a\,t}\cos(\omega_o t)\,u(t)$	$\frac{s+a}{(s+a)^2+\omega_o^2}$	$\mathrm{Re}(s) > -a$
$\mathrm{e}^{-a\,t}\sin(\omega_o t)\,u(t)$	$\frac{\omega_o}{(s+a)^2+\omega_o^2}$	$\mathrm{Re}(s) > -a$

Note: a is assumed real.

Laplace transform properties

$x(t)$	$X(s)$
$a\,x(t)+b\,g(t)$	$a\,X(s)+b\,G(s)$
$x(t) * g(t)$	$X(s)\,G(s)$
$\frac{dx(t)}{dt}$	$s\,X(s)$
$x(t-t_o)$	$\mathrm{e}^{-s\,t_o}\,X(s)$

Fourier series

If $x(t)$ is periodic with period T then

$$x(t) = \sum c(k)\,\mathrm{e}^{\mathrm{j}k\,\omega_o\,t}$$

where

$$\omega_o = \frac{2\,\pi}{T}$$

and

$$c(k) = \frac{1}{T}\int_{\langle T\rangle} x(t)\,\mathrm{e}^{-\mathrm{j}k\omega_o t}\,dt$$

selected Fourier transform pairs

$x(t)$	$X^f(\omega)$
$x(t)$	$\int x(t)\,\mathrm{e}^{-\mathrm{j}\omega t}\,dt$ (def.)
$\frac{1}{2\pi}\int X^f(\omega)\,\mathrm{e}^{\mathrm{j}\omega t}\,d\omega$	$X^f(\omega)$
$\delta(t)$	1
1	$2\,\pi\,\delta(\omega)$
$u(t)$	$\pi\,\delta(\omega)+\frac{1}{\mathrm{j}\omega}$
$\mathrm{e}^{\mathrm{j}\omega_o t}$	$2\,\pi\,\delta(\omega-\omega_o)$
$\cos(\omega_o\,t)$	$\pi\,\delta(\omega+\omega_o)+\pi\,\delta(\omega-\omega_o)$
$\sin(\omega_o\,t)$	$\mathrm{j}\,\pi\,\delta(\omega+\omega_o)-\mathrm{j}\,\pi\,\delta(\omega-\omega_o)$
$\frac{\omega_o}{\pi}\ \mathrm{sinc}\left(\frac{\omega_o}{\pi}t\right)$	ideal LPF cut-off frequency ω_o
symmetric pulse width T, height 1	$\frac{2}{\omega}\ \sin\left(\frac{T}{2}\,\omega\right)$
impulse train period T, height 1	impulse train period, height $\omega_o = \frac{2\pi}{T}$

Fourier transform properties

$x(t)$	$X^f(\omega)$
$a\,x(t)+b\,g(t)$	$a\,X^f(\omega)+b\,G^f(\omega)$
$x(a\,t)$	$\frac{1}{\lvert a\rvert}\,X\left(\frac{\omega}{a}\right)$
$x(t) * g(t)$	$X^f(\omega)\,G^f(\omega)$
$x(t)\,g(t)$	$\frac{1}{2\pi}\,X^f(\omega) * G^f(\omega)$
$x(t-t_o)$	$\mathrm{e}^{-\mathrm{j}t_o\omega}\,X^f(\omega)$
$x(t)\,\mathrm{e}^{\mathrm{j}\omega_o t}$	$X^f(\omega-\omega_o)$
$x(t)\ \cos(\omega_o t)$	$0.5\,X^f(\omega+\omega_o)+0.5\,X^f(\omega-\omega_o)$
$x(t)\ \sin(\omega_o t)$	$\mathrm{j}\,0.5\,X^f(\omega+\omega_o)-\mathrm{j}\,0.5\,X^f(\omega-\omega_o)$
$\frac{dx(t)}{dt}$	$\mathrm{j}\,\omega\,X^f(\omega)$