

ECE 3054: Signals, Systems, and Transforms

Lab Manual

1. The lab will meet every week.
2. Be sure to review the lab ahead of the lab session. Please ask questions of the TA's if you need some help, but also, please prepare in advance for the labs by reading the lab closely.
3. Your activity, participation, and progress *during* the lab session will be part of your lab grade. (Don't browse the web during lab!)
4. The lab consists of computer-based exercises. You are required to bring your laptop computer to lab with MATLAB installed. In case you do not have MATLAB installed on your laptop computer you can go to the laptop help desk to have it installed. You will need the *Signal Processing Toolbox*.
5. Labs and the data files required for some labs will be available on the web at:
<http://eeweb.poly.edu/iselesni/EE3054/lab/>
6. A lab report for each lab (except for Lab 1) will be due the following week in lab. Your lab report should include relevant code fragments, figures, answers to discussion questions in the lab, and explanations. No report is required for Lab 1.
7. There will be at least **two** paper/pencil **quizzes** related to the lab during the semester. The first lab quiz will be early in the semester and will focus on MATLAB usage only. The second lab quiz will be later in the semester and will cover concepts from the labs and lecture and MATLAB programming.

At the end of this lab manual, there is an example quiz 1. You should be able to answer all the questions on this example quiz before taking the first MATLAB quiz.
8. *The earlier in the semester you become comfortable with MATLAB the better.* The first lab introduces you to MATLAB. Several MATLAB tutorials are available at:

<http://eeweb.poly.edu/iselesni/Matlab/>

You should go through the tutorial *Getting Started with MATLAB*. Be sure to download this tutorial to your computer as soon as possible. This tutorial can also be downloaded for free from the Mathworks website:

<http://www.mathworks.com/>

Other documentation can be obtained at:

<http://www.mathworks.com/access/helpdesk/help/helpdesk.shtml>

9. In preparation for some of the labs you should read the additional lab notes. These additional notes for the lab are contained in the pdf file: "AddLabNotes.pdf" available on the lab web site.

Lab 1: Introduction to Matlab

1. Creating vectors

- (a) Generate the following vectors:

$$A = [1 \ 0 \ 4 \ 5 \ 3 \ 9 \ 0 \ 2]$$

$$a = [4 \ 5 \ 0 \ 2 \ 0 \ 0 \ 7 \ 1]$$

Be aware that Matlab are case sensitive. Vector **A** and **a** have different values.

- (b) Generate the following vectors:

$$B = [A \ a]$$

$$C = [a, A]$$

Concatenation is the process of joining small matrices to make bigger ones. In fact, you made your first matrix by concatenating its individual elements. The pair of square brackets, `[]`, is the concatenation operator.

- (c) Generate the following vectors using function **zeros** and **ones**:

$$D = [0 \ 0 \ 0 \ \dots \ 0] \text{ with fifty } 0\text{'s.}$$

$$E = [1 \ 1 \ 1 \ \dots \ 1] \text{ with a hundred } 1\text{'s.}$$

- (d) Generate the following vectors using the colon operator

$$F = [1 \ 2 \ 3 \ 4 \ \dots \ 30]$$

$$G = [25 \ 22 \ 19 \ 16 \ 13 \ 10 \ 7 \ 4 \ 1]$$

$$H = [0 \ 0.2 \ 0.4 \ 0.6 \ \dots \ 2.0]$$

The colon, `:`, is one of Matlab's most important operators.

2. Operate with the vectors

$$V1 = [1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 0]$$

$$V2 = [0.3 \ 1.2 \ 0.5 \ 2.1 \ 0.1 \ 0.4 \ 3.6 \ 4.2 \ 1.7 \ 0.9]$$

$$V3 = [4 \ 4 \ 4 \ 4 \ 3 \ 3 \ 2 \ 2 \ 2 \ 1]$$

- (a) Calculate, respectively, the sum of all the elements in vectors **V1**, **V2**, and **V3**.

- (b) How to get the value of the fifth element of each vector?

What happens if we execute the command `V1(0)` and `V1(11)`?

Remember if a vector has **N** elements, their subscripts are from 1 to **N**.

(c) Generate a new vector **V4** from **V2**, which is composed of the first five elements of **V2**.

Generate a new vector **V5** from **V2**, which is composed of the last five elements of **V2**.

(d) Derive a new vector **V6** from **V2**, with its 6th element omitted.

Derive a new vector **V7** from **V2**, with its 7th element changed to 1.4.

Derive a new vector **V8** from **V2**, whose elements are the 1st, 3rd, 5th, 7th, and 9th elements of **V2**

(e) What are the results of

`9-V1`

`V1*5`

`V1+V2`

`V1-V3`

`V1.*V2`

`V1*V2`

`V1.^2`

`V1.^V3`

`V1^V3`

`V1 == V3`

`V1>6`

`V1>V3`

`V3-(V1>2)`

`(V1>2) & (V1<6)`

`(V1>2) | (V1<6)`

`any(V1)`

`all(V1)`

3. Using Matlab help system, click on

```
Help -> MATLAB help
```

or type `helpdesk` to can open the help files. For description of a single function or command, type

```
help command_name
```

on the command line, or use 'search' in the help window.

For example, type

```
help plot
```

on the command line.

4. Flow control

What are the results of these sets of commands? Think them over and run them with Matlab to see if you are right.

```
(a) A = zeros(1,5);
    for n = 1:4
        for m = 1:3
            A = A + n*m;
        end
    end
    A
```

```
(b) B = [1 0];
    if (all(B))
        B = B + 1;
    elseif (any(B))
        B = B + 2;
    else
        B = B + 3;
    end
    B
```

```

(c) C = 7:2:22
    num = 0;
    while (all( C>0))
        C = C - 3;
        num = num + 1;
    end
    C
    num

```

(b) Situations under which loops can be avoided. Does the set of commands:

```

for i = 1:20
    H(i) = i * 5;
end

```

have the same result as:

```

H = 1:20;
H = H*5;

```

Does the set of commands:

```

for n = 1:100000
    x(n) = sin(n*pi/10);
end

```

have the same result as:

```

n = 1:100000;
x = sin(n*pi/10);

```

5. Compare a script and a function

(a) Write a script: In the main menu of Matlab, select

```

file -> new -> M-file

```

A new window will pop up. Input the following commands:

```
x = 1:5;
y = 6:10;
g = x+y;
```

and then save the file as `myscript.m` under the default path `matlab/work`

- (b) Write a function: Create a new `m` file following the procedure of above. Type in the commands:

```
function g = myfunction(x,y)
g = x + y;
```

and then save it as `myfunction.m`

- (c) Compare their usage

- (1) run the commands one by one:

```
myscript
x
y
g
z = myscript    (error?)
```

- (2) Run command `clear` to remove all variables from memory

- (3) Run the commands one by one:

```
x = 1:5;
y = 6:10;
myfunction    (error?)
z = myfunction(x,y)
g            (error?)
a = 1:10;
b = 2:11;
myfunction(a,b)
```

Can you explain what is going on here?

Lab 2: Basic Plotting of Signals

Using MATLAB, make plots of the signals below. Put your code in a Matlab *script* file so you can rerun it from the Matlab command after you make revisions to your file.

Use the `subplot` command to put several plots on the same page. Print out the plots and turn them in with your code. Use `help subplot` to find out how to use the command.

Use the `plot` command to plot continuous-time signals. Use the `stem` command to plot discrete-time signals. Use `help plot` and `help stem` to find out how to use these commands. Our convention is that $f(t)$ denotes a continuous-time signal, and that $f(n)$ denotes a discrete-time signal.

1. Plotting Continuous-Time Signals

For the following: Run the following three lines and explain why the plots are different.

```
t = 0:2*pi; plot(t,sin(t))
```

```
t = 0:0.2:2*pi; plot(t,sin(t))
```

```
t = 0:0.02:2*pi; plot(t,sin(t))
```

For the last graph, add a title and axis labels with:

```
title('My Favorite Function')
```

```
xlabel('t (Seconds)')
```

```
ylabel('y(t)')
```

Change the axis with:

```
axis([0 2*pi -1.2 1.2])
```

Put two plots on the same axis:

```
t = 0:0.2:2*pi; plot(t,sin(t),t,sin(2*t))
```

Produce a plot with out connecting the points:

```
t = 0:0.2:2*pi; plot(t,sin(t),'.')
```

Try the following command:

```
t = 0:0.2:2*pi; plot(t,sin(t),t,sin(t),'r.')
```

What does the `r` do?

2. Plotting Discrete-Time Signals

Use `stem` to plot the discrete-time step-function:

```
n = -10:10;  
f = n >= 0;  
stem(n,f)
```

Make stem plots of the following signals. Decide for yourself what the range of n should be.

$$f(n) = u(n) - u(n - 4)$$

$$g(n) = n \cdot u(n) - 2(n - 4) \cdot u(n - 4) + (n - 8) \cdot u(n - 8)$$

$$x(n) = \delta(n) - 2\delta(n - 4)$$

$$y(n) = (0.9)^n (u(n) - u(n - 20))$$

$$v(n) = \cos(0.12\pi n) u(n)$$

3. The conv Command

Use `help conv` to find out how to use the `conv` command.

Let

$$f(n) = u(n) - u(n - 4)$$

$$g(n) = n \cdot u(n) - 2(n - 4) \cdot u(n - 4) + (n - 8) \cdot u(n - 8).$$

Make stem plots of the following convolutions. Use the MATLAB `conv` command to compute the convolutions.

(a) $f(n) * f(n)$

(b) $f(n) * f(n) * f(n)$

(c) $f(n) * g(n)$

(d) $g(n) * \delta(n)$

(e) $g(n) * g(n)$

Comment on your observations: Do you see any relationship between $f(n) * f(n)$ and $g(n)$? Compare $f(n)$ with $f(n) * f(n)$ and with $f(n) * f(n) * f(n)$. What happens as you repeatedly convolve this signal with itself?

Use the commands `title`, `xlabel`, `ylabel` to label the axes of your plots.

4. Plotting a Sampled-Signal.

Suppose the continuous-time signal $x(t)$ is sampled with a sampling period of 0.3 seconds between each sample. Suppose also that the first sample is at $t = 0$ and that a total of 12 samples are collected. The table below lists the samples of $x(t)$ that were collected.

sample number	$x(t)$
0	6.0
1	-1.3
2	-8.0
3	-11.7
4	-11.0
5	-6.0
6	1.3
7	8.0
8	11.7
9	11.0
10	6.0
11	-1.3

Using the `plot` command, make a plot of this signal. The horizontal axis should be labeled: `TIME (SECONDS)`. Make sure that the horizontal time axis shows the correct time for each sample: 0, 0.3, 0.6, ..., 3.3.

Lab 3: Smoothing Data and Difference Equations

1. Convolution of Non-Causal Signals

Use Matlab to make stem plots of the following signals. Decide for yourself what the range of n should be. Note that both of these signals start to the left of $n = 0$.

$$f(n) = 3\delta(n+2) - \delta(n-1) + 2\delta(n-3)$$

$$g(n) = u(n+4) - u(n-3)$$

Next, use Matlab to make a stem plot of $x(n) = f(n) * g(n)$.

Also: plot the signals by hand without relying on Matlab and check that you get the same result as your Matlab plot (not to turn in).

To turn in: The plots of $f(n)$, $g(n)$, $x(n)$, and your Matlab commands to create these signals and plots.

2. Smoothing Data

Save the data file `DataEOG.txt` from the course website. Load the data into Matlab using the command `load DataEOG.txt`. Type `whos` to see your variables. One of the variables will be `DataEOG`. For convenience, rename it to `x` by typing: `x = DataEOG`; This signal comes from measuring electrical signals from the brain of a human subject.

Make a stem plot of the signal $x(n)$. You will see it doesn't look good because there are so many points. Make a plot of $x(n)$ using the `plot` command. As you can see, for long signals we get a better plot using the `plot` command. Although discrete-time signals are most appropriately displayed with the `stem` command, for *long* discrete-time signals (like this one) we use the `plot` command for better appearance.

Create a simple impulse response for an LTI system:

```
h = ones(1,11)/11;
```

Make a stem plot of the impulse response.

Find the output signal $y(n)$ of the system when $x(n)$ is the input (using the `conv` command).

Make plot of the output $y(n)$.

- What is the effect of the convolution in this example?
- How is the length of $y(n)$ related to the length of $x(n)$ and $h(n)$?

(c) Plot $x(n)$ and $y(n)$ on the same graph. What problem do you see? Can you get $y(n)$ to “line up” with $x(n)$?

(d) Use the following commands:

```
y2 = y;  
y2(1:5) = [];  
y2(end-4:end) = [];
```

What is the effect of these commands? What is the length of $y2$? Plot x and $y2$ on the same graph. What do you notice now?

(e) Repeat the problem, but use a different impulse response:

```
h = ones(1,31)/31;
```

What should the parameters in part (d) be now?

(f) Repeat the problem, but use

```
h = ones(1,67)/67;
```

What should the parameters in part (d) be now?

Comment on your observations.

To turn in: The plots, your Matlab commands to create the signals and plots, and discussion.

3. Difference Equations

Suppose a system is implemented with the difference equation:

$$y(n) = x(n) + 2x(n-1) - 0.95y(n-1)$$

Write your own Matlab function, `mydiffEq`, to implement this difference equation using a `for` loop. (Type `help for` to see how to use the `for` loop.) If the input signal is N -samples long ($0 \leq n \leq N-1$), your program should find the first N sample of the output $y(n)$ ($0 \leq n \leq N-1$). Remember that Matlab indexing starts with 1, not 0, but don't let this confuse you.

Use $x(-1) = 0$ and $y(-1) = 0$.

To see how to write a Matlab function see the *Matlab Getting Started Guide*. This tutorial to Matlab is very useful. It is available on the web — see the link on the course webpage.

(a) Is this system linear? Use your Matlab function to confirm your answer:

```
y1 = mydiffEq(x1)
```

```
y2 = mydiffEq(x2)
y3 = mydiffEq(x1+2*x2)
```

Use any signals x_1 , x_2 you like.

- (b) Is this system time-invariant? Confirm this in Matlab (how?).
- (c) Compute and plot the impulse response of this system. Use $x = [1, \text{zeros}(1,100)]$; as input.
- (d) Define $x(n) = \cos(\pi n/8) [u(n) - u(n - 50)]$. Compute the output of the system in two ways:
- (1) $y(n) = h(n) * x(n)$ using the `conv` command.
 - (2) Use your function to find the output for this input signal.
- Are the two output signals you compute the same?

- (e) Write a new Matlab function for the system with the difference equation:

$$y(n) = x(n) + 2x(n - 1)$$

Find and plots the impulse response of this system. What is the difference between the two impulse responses?

- (f) Write a new Matlab function for the system with the difference equation:

$$y(n) = x(n) + 2x(n - 1) - 1.1y(n - 1)$$

Find and plots the impulse response of this system. What do you find? Discuss your observations.

To turn in: The plots, your Matlab commands to create the signals and plots, and discussion.

Lab 4: Complex Poles

Before you begin this lab, please read the notes on *Partial Fraction Expansion*. These notes show how to use the Matlab `residue` command.

Given the transfer function $H(z)$ of a causal discrete-time LTI system, how do you find the impulse response $h(n)$? In this lab, we cover three ways get the impulse response from the transfer function. For the following problems, please use the following transfer function,

$$H(z) = \frac{1 - 2.5z^{-1}}{1 - z^{-1} + 0.7z^{-2}}.$$

1. What is the difference equation that implements this system? In Lab 3 you wrote a Matlab function that implements a difference equation. Modify that function so that it implements the difference equation for the system considered here. Use your new program to numerically compute the impulse response of this system. Once you have computed the impulse response, make a stem plot of it. By the way, Matlab has a built in command `filter` that implements a difference equation. Use the `help` command to find out how to use the `filter` command. Use the `filter` command to compute the impulse response. Does it agree with the one computed using your own program? (It should!)
2. A second way to get $h(n)$ from $H(z)$ is to use partial fraction expansion. Partial fraction expansion can be performed in Matlab with the `residue` command. Use this command to perform partial fraction expansion on $H(z)/z$. Then you can write the impulse response as

$$h(n) = C_1 p_1^n u(n) + C_2 p_2^n u(n). \quad (1)$$

The four values C_1, C_2, p_1, p_2 are found using the `residue` command. For this example they will be complex! Use Equation (1) to compute in Matlab the impulse response,

```
n = -5:30;
h = C1 * p1.^n .* (n>=0) + ...
```

Note that even though C_1, C_2, p_1, p_2 are complex, the impulse response $h(n)$ is real-valued. (The imaginary parts cancel out.) Is this what you find? Make a stem plot of the impulse response you have computed using Equation (1). Verify that the impulse response is the same as the one you computed in the previous part.

3. In this part you are to write a formula for the impulse response that does not involve complex numbers. The impulse response you have found can also be written as

$$h(n) = A r^n \cos(\omega_o n + \theta_o) u(n). \quad (2)$$

This is a damped sinusoid. This formula is obtained from Equation (1) by putting the complex values C_1, C_2, p_1, p_2 into polar form:

$$\begin{aligned} C_1 &= R_1 e^{j\alpha_1} \\ C_2 &= R_2 e^{j\alpha_2} \\ p_1 &= r_1 e^{j\beta_1} \\ p_2 &= r_2 e^{j\beta_2}. \end{aligned}$$

Note about complex numbers: to put a complex number c into polar form ($c = r e^{j\alpha}$) we set $r = |c|$ and $\alpha = \angle c$. In Matlab, $|c|$ is computed with the command `abs(c)` and $\angle c$ is computed with the command `angle(c)`.

In part (b) you found the complex values C_1 , etc. Now find the real values R_1, α_1 , etc, by using the commands `abs` and `angle`. You should find that $R_2 = R_1, \alpha_2 = -\alpha_1, r_2 = r_1$, and $\beta_2 = -\beta_1$. Is this what you find? Therefore, the formula in Equation (1) becomes

$$\begin{aligned} h(n) &= R_1 e^{j\alpha_1} (r_1 e^{j\beta_1})^n u(n) + R_1 e^{-j\alpha_1} (r_1 e^{-j\beta_1})^n u(n) \\ &= R_1 e^{j\alpha_1} r_1^n e^{j\beta_1 n} u(n) + R_1 e^{-j\alpha_1} r_1^n e^{-j\beta_1 n} u(n) \\ &= R_1 r_1^n (e^{j(\beta_1 n + \alpha_1)} + e^{-j(\beta_1 n + \alpha_1)}) u(n) \\ &= 2 R_1 r_1^n \cos(\beta_1 n + \alpha_1) u(n). \end{aligned}$$

This finally has the form in Equation (2). The equation now contains no complex numbers. Although the derivation looks messy, each step uses a basic identity which you should know. The last step uses the complex form for cosine.

In Matlab, find the real constants in Equation (2) and use this equation to compute the impulse response,

```
n = -5:30;
h = A * r.^n .* ...
```

Also, make a stem plot of the impulse response. It should be exactly the same as what you got in the previous two parts.

4. Please note the following!

- (a) The frequency ω_o of the damped cosine in Equation (2) is exactly the angle of the pole p_1 .

- (b) The decay of the damped cosine depends on r which is exactly the absolute value (modulus) of the pole p_1 .

Therefore, the locations of the poles of an LTI system provides information about the form of the impulse response.

To make a diagram of the poles and zeros of the system, use the `zplane` command.

```
zplane(b,a)
```

where `b` and `a` are the same vectors used for the `filter` and `residue` commands. (Note: when using `zplane` the vectors `b` and `a` must be *row* vectors. If they are column vectors you must take the transpose of them to get row vectors.)

Make a diagram of the poles and zeros of the above system using `zplane`.

5. The system considered above was a very simple one. For additional practice, take the transfer function

$$H(z) = \frac{1 - 0.6 z^{-1}}{1 - 2.1 z^{-1} + 1.6 z^{-2} - 0.4 z^{-3}}.$$

For this system repeat the previous parts. For this system Equation (1) will have three components instead of two. Equation (2) will have an extra term:

$$h(n) = A r^n \cos(\omega_o n + \theta_o) u(n) + B p_3^n u(n).$$

To turn in: the plots you produced, the Matlab code to produce the plots and intermediate results (show your use of the `residue` and `filter` commands). Also include discussion of your observations, explanation of your steps, and derivation of the impulse response in Equation (1).

Lab 5: Frequency Responses

Consider a causal discrete-time LTI system implemented using the difference equation,

$$y(n) = 0.1x(n) - 0.1176x(n-1) + 0.1x(n-2) + 1.7119y(n-1) - 0.81y(n-2)$$

For convenience, the frequency response of the system, $H(e^{j\omega})$, is sometimes denoted $H^f(\omega)$.

1. What is the transfer function $H(z)$ of the system? (Not a Matlab question!)
2. Plot the magnitude of the frequency response $H^f(\omega)$ of the system using the Matlab command `freqz`:

```
>> [H,w] = freqz(b,a);  
>> plot(w,abs(H));
```

where `b` and `a` are appropriately defined.

3. When the input signal is

$$x(n) = \cos(0.1\pi n)u(n) \tag{3}$$

find the output signal $y(n)$ using the Matlab command `filter`. Use $n = 0, 1, 2, \dots, 100$. Make a stem plot of the input and output signals. Use `subplot(2,1,1)` for $x(n)$ and `subplot(2,1,2)` for $y(n)$. Comment on your observations. How could $y(n)$ be predicted from the frequency response of the system?

4. Lets find the exact value of $H^f(\omega)$ at $\omega = 0.1\pi$. First, express $H^f(0.1\pi)$ as

$$H^f(0.1\pi) = \frac{B(e^{j0.1\pi})}{A(e^{j0.1\pi})}$$

where $B(z)$ and $A(z)$ are polynomials. Second, use the `exp` command to find the complex number $z = e^{j0.1\pi}$. Third, evaluate $B(z)$ and $A(z)$ at the complex value $z = e^{j0.1\pi}$ using the Matlab command `polyval`. (Use `help` to see how to use this command.)

5. Recall that when the input signal $x(n)$ is

$$x(n) = \cos(0.1\pi n)$$

then the output signal is

$$y(n) = |H^f(0.1\pi)| \cos(0.1\pi n + \angle H^f(0.1\pi)).$$

(Assuming that the impulse response $h(n)$ is real.) Here is the general diagram:

$$\cos(\omega_o n) \longrightarrow \boxed{h(n)} \longrightarrow |H^f(\omega_o)| \cos(\omega_o n + \angle H^f(\omega_o))$$

Note that this input signal $x(n)$ starts at $n = -\infty$. (There is no $u(n)$ term.) However, we are interested here in the case when $x(n)$ starts at $n = 0$, namely when $x(n)$ is given by Equation (3). In this case, we have

$$\cos(\omega_o n) u(n) \longrightarrow \boxed{h(n)} \longrightarrow |H^f(\omega_o)| \cos(\omega_o n + \angle H^f(\omega_o)) u(n) + \text{Transients}$$

The transients die out, and the *steady-state* output signal is simply $|H^f(\omega_o)| \cos(\omega_o n + \angle H^f(\omega_o)) u(n)$.

In Matlab, create a stem plot of the steady-state output signal

$$s(n) = |H^f(0.1 \pi)| \cos(0.1 \pi n + \angle H^f(0.1 \pi))$$

Compare your plot of $s(n)$ with your plot of $y(n)$ you computed using the `filter` command. You can do this by plotting them on the same graph with

`plot(n,y,n,s)`

(`stem` does not make multiple plots on the same graph, so we use `plot` here.) You should find that $s(n)$ and $y(n)$ agree after a while. (After the transient response has died out.) Is this what you find? After the transient response has died out, the steady-state output signal $s(n)$ remains.

6. When the input signal is

$$x(n) = \cos(0.3 \pi n) u(n)$$

find the output signal $y(n)$ using the Matlab command `filter`. Use $n = 0, 1, 2, \dots, 100$. Make a stem plot of the input and output signals. Use `subplot(2,1,1)` for $x(n)$ and `subplot(2,1,2)` for $y(n)$. Comment on your observations. How could $y(n)$ be predicted from the frequency response of the system? What is the steady-state signal? Relate your explanation to the concept describe in the previous question.

7. Plot the poles and zeros of the transfer function using the Matlab command `zplane`:

`zplane(b,a)`

Comment on how the frequency response magnitude $|H^f(\omega)|$ can be predicted from the pole-zero diagram.

8. Some Simple Systems

For the causal discrete-time LTI systems implemented by each of the following difference equations plot the frequency response magnitude $|H^f(\omega)|$, the pole-zero diagram, and the impulse response.

$$y(n) = x(n) + 1.8y(n-1) - 0.9y(n-2) \quad (4)$$

$$y(n) = x(n) + 1.6y(n-1) - 0.72y(n-2) \quad (5)$$

$$y(n) = x(n) + 1.53y(n-1) - 0.9y(n-2) \quad (6)$$

$$y(n) = x(n) + 1.4y(n-1) + 0.72y(n-2) \quad (7)$$

$$y(n) = x(n) - 0.85y(n-1) \quad (8)$$

$$y(n) = x(n) - 0.95y(n-1) \quad (9)$$

Comment on your observations: How can you predict from the pole-zero diagram what the frequency response will look like and what the impulse response will look like? (Remember the partial fraction expansion procedure.)

Suppose you were given a diagram of each of the pole-zero diagrams, frequency responses, and impulse responses, but that they were out of order. Could you match each diagram to the others (without doing any computation)?

9. Placing the Zeros and Poles

In this part of the lab, you are to derive the difference equation of a system that has the poles and zeros you want it to have. Suppose you want a 3rd order LTI system with the following poles and zeros.

$$p_1 = 0.8 e^{j0.2\pi}, p_2 = 0.8 e^{-j0.2\pi}, p_3 = 0.7, z_1 = -1, z_2 = e^{j0.6\pi}, z_3 = e^{-j0.6\pi}$$

Use the `poly` command in Matlab to find $B(z)$ and $A(z)$ from the zero and poles. This gives `b` and `a` from which you can write down the difference equation. Also make a stem plot of the impulse response and frequency response magnitude. You can use `zplane` to plot the pole and zeros of the system. This way you can verify that the system you derived has the poles and zeros you designed it to have.

Now change the poles p_1 and p_2 to

$$p_1 = 0.98 e^{j0.2\pi}, p_2 = 0.98 e^{-j0.2\pi},$$

find the new difference equation, and plot the impulse response and frequency response magnitude.

How does this change to the position of the poles affect the nature of the impulse response of the system and the frequency response?

To turn in: The plots you produced, your Matlab commands to produce the plots, and your discussion of your observations.

Lab 6: Recursive Digital Filters

Please carefully read the notes on *Recursive Digital Filters* before the lab.

1. Use the Matlab function `ellip` to obtain a 3^{rd} order digital filter with pass-band ripple $\delta_p = 0.01$, stop-band ripple $\delta_s = 0.01$, and cut-off frequency 0.4π . [That means, use `Wn = 0.4` in `ellip`.] You will need to convert δ_p, δ_s to R_p and R_s . Make a plot of the magnitude of the frequency response, on a linear scale and in dB, and of the impulse response. Also make a plot of the pole-zero diagram (using `zplane`).
2. Repeat the previous problem, but instead use a filter of order 4.
3. Repeat the previous problem, but obtain instead a filter of order 6. Comment on your observations. How does the frequency response of the elliptic filter improve as the filter order is increased? How does the impulse response change?
4. Read the full `help` file for `ellip`. This command can be used to obtain high-pass filters, band-pass filters, and band-stop filters as well. Design a discrete-time high-pass filter and plot the frequency response, impulse response, and pole-zero diagram.
5. Design a discrete-time bandpass filter and plot the frequency response, impulse response, and pole-zero diagram.
6. Other types of IIR digital filters can be design with Matlab. The Butterworth and Chebyshev filters are commonly used filter types. These filters can be designed in Matlab using the commands `butter`, `cheby1`, `cheby2`. Use the `help` command to investigate these filters. Design a Butterworth low-pass digital filter of order 4 using `butter`, and make the corresponding graphs.
7. Design a Chebyshev-I low-pass IIR digital low-pass filter of order 4 using `cheby1`, and make the corresponding graphs.
8. Design a Chebyshev-II low-pass IIR digital low-pass filter of order 4 using `cheby2`, and make the corresponding graphs.

To turn in: Graphs of the frequency responses, impulse responses, and pole-zero diagram and your comments: In what ways do the four types of IIR filters differ (`ellip`, `butter`, `cheby1`, `cheby2`)? How do they differ in the frequency response and pole-zero plots? What happens as you increase the order of these filters?

Lab 7: Noise Reduction

Matlab comes with a speech data set of someone saying ‘Matlab’. The Matlab command `load mtlb` will load this data set. If your computer supports sound, you can listen to the original signal with the command `soundsc(mtlb)`. [Type `help soundsc` to find out how this command works.] The speech data set was obtained with a sampling frequency of $F_s = 7418$ Hz. The sampling period is $T_s = 1/F_s$.

The signal can be plotted with the following commands.

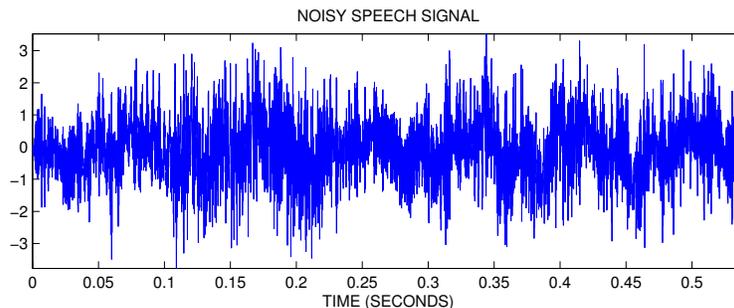
```
>> clear, close all
>> load mtlb
>> who
```

Your variables are:

```
Fs      mtlb
```

```
>> L = length(mtlb);
>> plot([1:L]/Fs,mtlb)
>> axis tight
>> xlabel('TIME (SECONDS)')
```

The data set `NoisySpeech.txt` on the class webpage, shown in the following figure, contains the `mtlb` data set corrupted by additive noise.



We will call the noisy signal $x(n)$. You are to design a low-pass filter to filter out the noise. You will try different low-pass filters until you find one that works well.

1. Download the data from the course webpage and make a plot of the data. You should get the same figure as shown above.

To obtain the data file, go to the course webpage and save the file `NoisySpeech.txt`, then type `load NoisySpeech.txt` on the Matlab command line. (You may need to change your Matlab working directory with the `cd` command to where-ever you have saved the file. Alternatively, you can append the directory to the Matlab path with the command `path`).

Typing `load NoisySpeech.txt` on the Matlab command line will create a vector called `NoisySpeech`. For convenience, type `x = NoisySpeech`; on the command line so that you can use `x` instead of the longer variable name. The command `plot([1:L]/Fs,x)` will then display the plot above.

If your computer supports sound, you can listen to the noisy signal $x(n)$ with the command `soundsc(x)`.

2. Use the `dtft` command as shown to plot the magnitude of the spectrum of the signal `mtlb`. What frequencies are predominant in the speech signal?

The `dtft` is not a built in Matlab command, you must download it from the course webpage. DTFT stands for the Discrete-Time Fourier Transform.

```
>> [M,f] = dtft(mtlb,1/Fs);
>> plot(f,M)
>> xlabel('FREQUENCY (Hz)')
>> title('SPECTRUM of MTLB')
```

3. As in the previous part, use the `dtft` command to plot $|X^f(\omega)|$, the spectrum of the noisy signal $x(n)$.

Can you identify which part of the spectrum corresponds to the noise and which part corresponds to the speech signal?

Explain how a low-pass filter can remove the high frequency noise.

From the spectrum can you estimate what the cut-off frequency of the lowpass filter should be?

4. Design an appropriate elliptical low-pass filter with the `ellip` command. Use the `filter` command to filter the noisy signal $x(n)$ with the lowpass filter you design.

Try different cut-off frequencies and different values for the stop-band ripple and the pass-band ripple. Can you remove most of the noise from the noisy signal?

Is the noise reduction quality affected more by the size of passband ripple or by the size of the stopband ripple? (What is more important — that the passband be very close to 1 or that the stopband be very close to 0? You should try different filter designs and listen to the results to find out.)

Lab 8: Synthesizing the Sound of a Plucked String

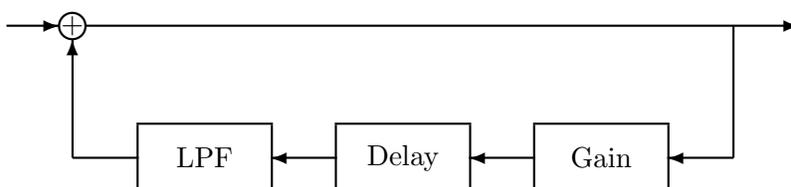
There are several methods for electronically producing the sound of musical instruments. One method is to record the sound, sample it, and save the samples. This method is computationally simple and produces naturalistic sounds, however, it requires a lot of memory and lacks flexibility: For each note, all the samples must be stored in memory. In addition, it is not possible to tune the notes.

A second method to simulate the sound of a musical instrument is to model the sound as a sum of cosine signals. This method requires very little memory — only the frequency and amplitude of each cosine signal. However, this method usually produces somewhat artificial sounds.

A third method is based on physically modelling the instrument using an appropriately designed system. Methods based on physical modeling can sound realistic, need very little memory, are tunable, and can be efficiently implemented. In this lab, we investigate a system for simulating the sound of a plucked string.

K. Karplus and A. Strong described a very simple system for generating a signal which sounds like a plucked string. This system can be used to synthesize the sound of a guitar. They published their method in the paper “Digital Synthesis of Plucked-String and Drum Timbres” in the *Computer Music Journal* in 1983.

The Karplus-Strong system uses this system:



This system contains three components in a feedback loop: a lowpass filter (LPF), a delay, and a gain. We will use the simplest lowpass filter and the simplest delay.

1. The simplest digital lowpass filter is the two-point averager. Its transfer function is $H(z) = 0.5 + 0.5z^{-1}$.
2. The simplest delay is an integer delay by N samples. Its transfer function is $D(z) = z^{-N}$.
3. The transfer function for the gain is $G(z) = K$. For the system to be stable, it is required that $K < 1$.

$$\begin{aligned} \text{LPF: } & H(z) = 0.5 + 0.5 z^{-1} \\ \text{Delay: } & D(z) = z^{-N} \\ \text{Gain: } & G(z) = K \end{aligned}$$

Question: What is the transfer function of the Karplus-Strong system pictured above? How many poles does it have?

Question: What is the difference equation that implements this system?

Question: We will use the command

```
y = filter(b,a,x);
```

to implement this system. What should **a** and **b** be?

To simulate the sound of a plucked string, the input to the system should be an N -point random signal. In Matlab, the following command can be used to make the input signal.

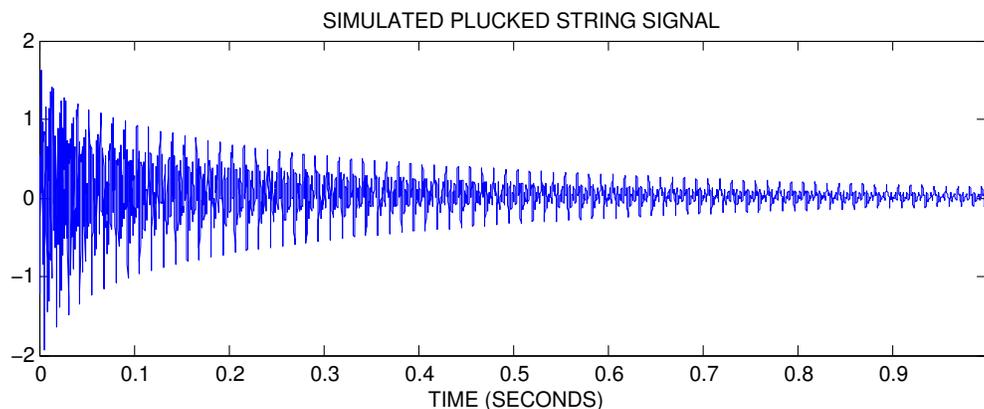
```
x = [randn(1,N) zeros(1,L)];
```

Then when we use `y = filter(b,a,x)` we will get a signal **y** that is $N + L$ points long.

We can then listen to the result with the command `soundsc(y,Fs)`. Where **Fs** is the sampling frequency. For this lab use **Fs** = 8000.

Question: With **Fs** = 8000 what should **N+L** be if we want the output signal to be 1 second in duration?

When $N = 100$, $K = 0.98$, the system produces the following output signal. Because the input signal is random for the first N points, the output signal will be a bit different each time this method is used.



In Matlab: Write a Matlab program to simulate the sound of a plucked-string using the Karplus-Strong system above. Use the values $N = 100$, $K = 0.98$. Choose L so that the output signal is one second in duration. Use the `filter` command. Listen to the output signal using the `soundsc` command. Does it sound like a plucked string?

Zoom in on the signal (magnify a small portion of it). Does it look almost periodic?

Run your program several times. Does it sound exactly the same each time?

In Matlab: Plot the frequency response of the system using the `freqz` command. To ensure that the plot captures the details of the frequency plot, you can use `freqz` with a third input argument, for example:

```
[H,w] = freqz(b,a,2^16)
plot(w/pi*Fs/2,abs(H))
```

This will give a denser frequency axis grid.

Question: From the frequency response, how could you have predicted that the output signal of the Karplus-Strong system is nearly periodic? (Note: the spectrum of a periodic signal is a line spectrum.)

We can tune the sound by changing the system parameters.

In Matlab: Run the system again, using different values of K . Plot the output signal, listen to the signal, and plot the frequency response. When you change K what effect does it have on the output signal and the frequency response?

In Matlab: Run the system again, using different values of N . Plot the output signal, listen to the signal, and plot the frequency response. When you change N what effect does it have on the output signal and the frequency response?

Note: The fundamental frequency F_o of the sound produced by this system will depend on the sampling frequency F_s , the delay N , and the delay incurred by the lowpass filter. It is

$$F_o = \frac{F_s}{N + d}$$

where d is the delay of the lowpass filter. The two-point averager has a delay of $d = 0.5$.

In Matlab: In the above description, we are driving the system with a burst of noise (that means, the input to the system is a short random signal). Suppose the system is driven by only an impulse. Run the system with the input $\mathbf{x} = [1 \text{ zeros}(1,L)]$. (The output will be just the impulse response of the system.) How does this effect the sound quality of the output signal? Does it sound more or less natural? Take a close look at the output signal, what do you notice?

In Matlab: Suppose the lowpass filter is taken out of the Karplus-Strong system. If the feedback loop of this system contains only the gain K and the delay z^{-N} then it is called a *comb* filter. For the comb filter, what is the difference equation and transfer function? Run the system without the lowpass filter component. Plot the output signal, listen to the signal, and plot the frequency response. How does removing the lowpass filter effect the sound and the frequency response of the system? Why is this system called a comb filter?

Notes:

1. The frequency of the sound produced by the Karplus-Strong system can not be continuously varied. The delay N must be an integer, so only a discrete set of fundamental frequencies F_o can be simulated with this model. To overcome this limitation we can introduce a *fractional delay* in the feedback loop.
2. If you allow the delay in the feedback loop to be a *time-varying* delay, then the frequency of the sound can vary over the duration of the note. Time-varying systems add flexibility.
3. You can learn more about simulating musical instruments using signals and systems on the web.

http://www.harmony-central.com/Synth/Articles/Physical_Modeling/

Lab 9: Stable Inverses

1. Suppose that a causal LTI discrete-time system has the impulse response

$$h_1(n) = 0.5^n u(n) \quad (10)$$

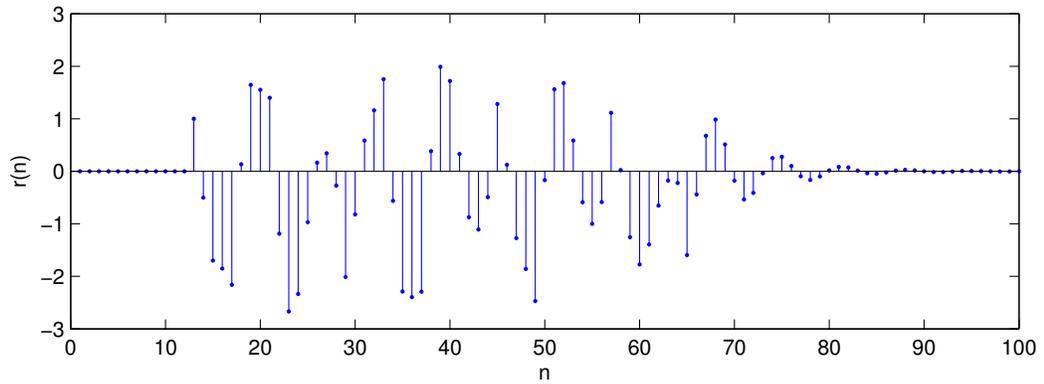
and that an anti-causal LTI discrete-time system has the impulse response

$$h_2(n) = 3^n u(-n - 1). \quad (11)$$

- (a) If these two systems are connected in cascade, what is the transfer function $H(z)$ and ROC of the total system?
- (b) Find the impulse response $h(n)$ of the total system, show your derivation, and make a stem plot of $h(n)$ using Matlab.
- (c) Consider the stable inverse $G(z) = 1/H(z)$. Find the ROC of $G(z)$ and $g(n)$. Make a stem plot $g(n)$ using Matlab. Verify in Matlab that it is an inverse by convolving $g(n)$ and $h(n)$ to get $\delta(n)$. (You will need to truncate h and g first as they are infinite in length.)
2. **Distortion removal.** A binary signal is sent over an imperfect channel which introduces distortion. The channel can be modeled as a discrete-time LTI system with the transfer function

$$H(z) = \frac{1 - 2.5z^{-1} + z^{-2}}{1 - z^{-1} + 0.7z^{-2}}.$$

- (a) Determine the impulse response $g(n)$ of the **stable** inverse of $H(z)$. You may use the `residue` command in Matlab.
- (b) Using Matlab, make a stem plot of $h(n)$, $g(n)$, and $h(n) * g(n)$. Check that you get an impulse.
- (c) Using Matlab (`freqz`) plot $|H^f(\omega)|$ and $|G^f(\omega)|$, the magnitudes of the frequency responses. Also plot the product $|H^f(\omega) \cdot G^f(\omega)|$. What do you expect to get for the product? Explain.
- (d) The binary signal $x(n)$ is sent through the channel and the received signal $r(n)$ is shown in the figure.



The signal $r(n)$ is contained in the accompanying file `DistortedSignal.txt`, but not $x(n)$. Import $r(n)$ and make plot it using Matlab. Filter $r(n)$ with the impulse response $g(n)$ and make a plot of the new signal. Has the distortion been removed?

- (e) Suppose the frequency response $H^f(\omega)$ were equal to zero for some frequency ω_o . Would that make the design of an inverse system more difficult? Explain.

Lab 10: Echo Cancellation

In this lab you will work on the problem of removing an echo from a recording of a speech signal. An echo has a couple of parameters: The first parameter is the delay: t_o seconds (or N samples). The second parameter is the amplitude of the echo: α .

The lab has the following parts:

1. **Create the Echo Signal.** You will add an echo to a speech signal.
2. **Remove the Echo.** You will remove an echo from a speech signal. In this part you know what the echo parameters t_o and α are.
3. **Estimating Unknown Echo Parameters.** You will try to determine the echo parameters t_o and α from a speech signal, and then remove the echo.

The lab consists of the following activities and questions:

1. Create the Echo Signal

Load the ‘matlab’ speech signal with the command

```
>> load mtlb
```

Then type `who` to see your variables. You can listen to the signal `mtlb` with the command `soundsc(mtlb,Fs)`, provided your computer has a sound card. You should hear the phrase ‘matlab’. For convenience, lets redefine this to be `x` by typing `x = mtlb;`

Make a plot of the signal `x` versus time (in SECONDS). You will need the sampling frequency `Fs` which is loaded when you typed `load mtlb`. You should find that the speech signal is about 0.55 seconds in duration.

2. How do you find the exact duration of this signal in seconds?
3. We can create an echo by filtering this signal with the following difference equation:

$$y(n) = x(n) + \alpha x(n - N)$$

where $x(n)$ is the echo-free speech signal, which has been delayed by N samples and added back in with its amplitude scaled by α . This is a reasonable model for an echo resulting from the signal reflecting off an absorbing surface like a wall. The delay parameter t_o in seconds is N/F_s where F_s is the sampling frequency. (Can you explain why?) What is the impulse response of this system?

4. Lets add an echo to the signal with a delay of $t_o = 0.12$ seconds and an amplitude of $\alpha = 0.8$. What is the delay N in samples? Based on the difference equation, create an echo version of the signal \mathbf{x} using the command $\mathbf{y} = \text{conv}(\mathbf{h}, \mathbf{x});$. What is \mathbf{h} ?

Listen to the output of the difference equation. Do you hear the echo?

5. **Remove the Echo.**

The difference equation above for modeling the echo is an LTI system. To remove the echo, we need to implement the inverse system. What is the impulse response of the inverse system? What is the duration of this impulse response?

6. For the echo parameters you used above, plot the impulse response of the inverse system. Make the horizontal axis in your plot in units of SECONDS.
7. Implement the inverse of the echo system to perform *echo cancellation*. You can use the command $\mathbf{g} = \text{filter}(\mathbf{b}, \mathbf{a}, \mathbf{y})$. What are the vectors \mathbf{b} and \mathbf{a} ?

8. Listen to the result. Is the echo removed?

9. **Estimating Unknown Echo Parameters**

In a real problem, you will probably not know what the echo parameters t_o and α are. You will need to estimate them from the echo signal itself.

Suppose you are given the data $y(n)$ (which is corrupted by an echo) but suppose you do not know the value of the delay N . Determine a method of estimating N based on the autocorrelation function of $y(n)$. Let $r_{yy}(n)$ be the autocorrelation of $y(n)$,

$$r_{yy}(n) = y(n) * y(-n),$$

and let $r_{xx}(n)$ be the autocorrelation of $x(n)$,

$$r_{xx}(n) = x(n) * x(-n).$$

First, find a formula for the signal $r_{yy}(n)$ in terms of $r_{xx}(n)$ (this is not a Matlab question). To do this, use the expression you have for the impulse response together with properties of the convolution sum.

10. Compute in Matlab and plot $r_{xx}(n)$ vs time (in SECONDS). $r_{xx}(n)$ is the autocorrelation of the echo-free signal. To compute in Matlab the autocorrelation of a signal you can use the commands

```
>> rxx = conv(x,x(end:-1:1));
```

or the command `xcorr`. You should see that it has a peak in the middle.

11. For the signal and echo parameters above, plot $r_{yy}(n)$. Can you determine what the value of N is from the plot of $r_{yy}(n)$? (Look at the peaks of $r_{yy}(n)$.) Does it agree with the known value?
12. From the course website, there is a data file you can download for this lab. It contains a signal which is corrupted by an echo with unknown echo parameters t_o and α .
Can you estimate N for this data from its autocorrelation?
13. The amplitude α is harder to estimate than N . Once you have estimated N , try to remove the echo by trying out different values of α and listening to your result. What N and α do you find work best? Are you able to successfully remove the echo?
14. What if an echo has two components?

$$y(n) = x(n) + \alpha_1 x(n - N_1) + \alpha_2 x(n - N_2)$$

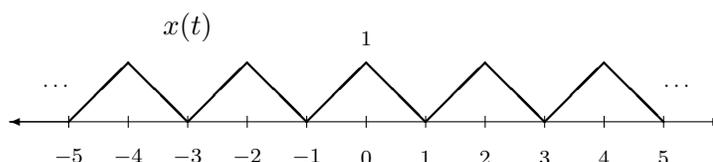
Discuss what system is required for echo cancellation. How would you find the echo parameters?

Lab 11: Fourier Series

1. Find the Fourier series coefficients $c(k)$ for the periodic signal

$$x(t) = |\cos(\pi t)|.$$

2. Find the Fourier series coefficients $c(k)$ for the periodic triangular pulse shown below.



3. (a) Run the Matlab program `fs1.m` to see the convergence of the Fourier series of the periodic pulse signal. The program is available on `my.poly.edu`.
- (b) Write a Matlab program `fs2.m` to generate the same plots but for the signal $x(t) = |\cos(t)|$. This entails replacing the line for `c0` and `ck` in `fs1.m`, with the formula from question 1.
- (c) Write a Matlab program `fs3.m` to generate the same plots but for the periodic triangular pulse.
- (d) For which signal does the Fourier series converge more rapidly, the periodic rectangular pulse signal or the periodic triangular signal? How could you predict your answer?
-

4. Numerical Computation of Fourier Series Coefficients

Sometimes the integration formula for the Fourier series coefficients is difficult to carry out. In these cases, the coefficients can be obtained numerically. Remember that an integral

$$\int f(t) dt \approx \Delta T \sum_m f(m \Delta T)$$

For example,

$$c(k) = \frac{1}{T} \int_{\langle T \rangle} x(t) e^{-jk\omega_0 t} dt$$

can be approximated as a sum:

$$c(k) \approx \frac{\Delta T}{T} \sum_{m=0}^{M-1} x(m \Delta T) e^{-jk\omega_0 m (\Delta T)}$$

The following Matlab program computes the first N Fourier series coefficients $c(k)$ numerically of the signal

$$x(t) = |\cos(\pi t)|$$

The program plots the line spectrum (a graph of the FS coefficients). The program also creates a periodic signal using the calculated coefficients — this verifies if the coefficients were computed correctly.

Run this program and verify that it works. It can be obtained from `my.poly.edu`

Change the program so that it computes the Fourier series coefficients of the periodic triangle wave shown above.

5. The Time-Shift Property

The time-shift property states that if a periodic signal is shifted, then the Fourier coefficients of that signal are modified according to the following rule: If

$$x(t) \quad \Longleftrightarrow \quad c(k)$$

then

$$x(t - t_o) \quad \Longleftrightarrow \quad e^{-j k \omega_o t_o} c(k)$$

Verify this property numerically using Matlab, by setting

$$g(t) = |\cos(\pi (t - 0.2))|.$$

Find the Fourier coefficients of $g(t)$ by first finding the Fourier coefficients of $x(t) = |\cos(\pi t)|$ and modifying those coefficients according to this rule. Then use the new coefficients to synthesize the periodic signal $g(t)$ by adding up complex exponentials. Are you able to confirm the time-shift property?

EE 3054: Signals, Systems, and Transforms

Example Matlab Quiz

No laptop, no notes, no documentation.

1. Given the following array `a`,

`a =`

```
1     4     2     4
7     5     9     2
-5    7    -2     0
```

determine the result of each of the following commands.

```
>> a(2, 3)
>> a(2, :)
>> a(6)
>> a(3, 2:end)
>> a(1:2, 4:-1:2)
>> a([2 2], [2 3])
>> a > 5
>> sum(a)
>> a(:)
>> [a(1,:), a(2,:)]
>> [a(1,:); a(2,:)]
```

2. What are the results of the following commands?

```
>> a = [5 2 3 5 8];
>> b = [9 2 5 0 8];
>> a == 5
>> a == b
```

3. What is the result of each of the following commands?

```
>> a = [zeros(3); ones(1,3)]
>> b = [zeros(3); ones(3,1)]
```

4. What is the result of the following command?

```
>> n = 0:0.5:3.2
```

5. What is the result of the following commands?

```
>> n = 2:7
>> n(2) = [];
>> n
```

6. What are all the results of the following commands?

```
>> a = [3 4; 7 8];
>> b = [1 0; 0 1];
>> a'
>> a - 1
>> a .* b
>> a * b
>> a | b
>> a & b
>> a .^ 2
>> a ^ 2
```

7. The following code fragment produces 3 graphs. Sketch each of the three graphs.

```
>> n = 0:7;
>> x = 2*n + 1;
>> stem(n,x)
>> plot(n,x)
>> y = (-1).^n;
>> plot(n,x,n,y)
```

8. Sketch the 3 graphs produced by the following code.

```
>> n = 0:10;
>> x = (n >= 1) - (n >=5);
>> stem(n,x)
>> h = (n == 5);
>> y = conv(h,x);
>> stem(0:20,y)
>> z = conv(x,x);
>> stem(0:20,z)
```

9. Write a short Matlab code that will plot a sinusoid of frequency 50 Hz for 10 cycles.

10. The file `kiwi.m` contains the following:

```
y = 5;
x = 6;
z = x + y;
```

The file `grape.m` contains the following:

```
function z = grape(x,y)
z = x + y;
```

What is the result of the following commands?

```
>> clear
>> x = 2;
>> y = 5;
>> kiwi
>> z
```

What is the result of the following commands?

```
>> clear
>> x = 2;
>> y = 5;
>> z = grape(x,y);
>> z
```

EE 3054: Signals, Systems, and Transforms

Lab Quiz 1 — Spring 2005

No laptop, no notes, no documentation.

1. Given the following array `a`,

```
a =  
    9    4    7    2  
    1    6    3    5  
    3   10    6    4
```

determine the result of each of the following commands.

```
>> a(2, 3)  
>> a(0, 2)  
>> a(5)  
>> a'  
>> a(:, [2 2 2])  
>> a(1:2:end, 1:2:end)  
>> a(end:-1:1, :)  
>> max(a)  
>> b = a; b([2 3],[1 4]) = [11 22; 33 44]; b  
>> b = a; b(:,2) = []; b  
>> log10([1 10 100 0.1])
```

2. What are the results of the following commands?

```
>> a = [9 4 7 2 8];  
>> a(2)  
>> a(1,2)  
>> a(2,1)  
>> a > 5  
>> find(a > 5)  
>> a * a
```

```
>> [a, a]
>> [M, k] = min(a); M, k
>> a(1:end-1)
>> a([1 1 1], :)
```

3. What is the result of each of the following commands?

```
>> a = [1+j, 1+2*j, 3, 4, 5*j];
>> k = find(imag(a)==0);
>> a(k)
```

4. What is the result of the following commands?

```
>> a = [];
>> for k = 5:-1:2
    a = [a, k];
end
>> a
```

5. What is the result of the following commands?

```
>> a = [-2 3];
>> b = [4 2 -1];
>> conv(a,b)
```

6. The following code fragment produces 3 graphs. Sketch each of the three graphs.

```
>> n = 2:0.5:4;
>> x = [3 1 2 0 3];
>> plot(n,x)
>> plot(x)
>> stem(n,x)
```

7. Write a MATLAB function called `over` that has one output and two inputs. The first input is a vector; the second input is a scalar. The output should be the sum of all those elements in the vector that exceed the scalar. For example,

```
>> over([5 1 3 6 9],4)
```

```
ans =
```

```
20
```

because the elements in the vector that are greater than 4 are: 5, 6, and 9, so we have $5 + 6 + 9 = 20$.

Your program should not use a `for` or `while` loop and it should not use an `if` statement.