Optimal Cross-layer Scheduling for Multicast in Multi-Channel Wireless Networks

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1 Capacity Model

We consider a generic multi-channel wireless network modeled by a *hyper graph* $G = (V, H)$, where V is the set of nodes, and H is the set of broadcast links (potentially operating at different channels). Each broadcast link $l \in H$ can be represented by a *hyper-arc* $l = \langle i, J \rangle$, with $i \in V$ being the transmitter and $J \subseteq H$ the set of intended receivers within i's broadcast range. To model selective broadcast and variable range/rate broadcast from a transmitter, we allow one node to have multiple hyper-arcs, each of which has a different subset of intended receivers. Due to interference between adjacent transmissions, not all broadcast links can be activated simultaneously. Let z_{iJ} be the transmission rate on link $\langle i, J \rangle$, and $\hat{\mathcal{Z}}$ be the set of rate vectors that can be scheduled at any given time. Through time sharing between different rate vectors in $\hat{\mathcal{Z}}$, the feasible link rate region of the whole network can be characterized by the convex hull of $\hat{\mathcal{Z}}$, $\mathcal{Z} \triangleq \mathcal{CH}(\hat{\mathcal{Z}})$. For clarity of presentation, we start with a single multicast session consisting of a source s and a set of receivers $T \subseteq V$. We will study the multiple multicast sessions case in Section 3. For a single multicast session, we are interested in the following questions:

1. What is the highest rate at which the source s can multicast data to all receivers in T?
2. How do we achieve the highest rate through joint data coding, routing, and link scheduling?

It has been shown recently that the optimal multicast rate can be achieved through network coding in general network topology. The problem can be cast into an information-flow based utility maximization. Specifically, let r_s be the multicast rate at source s, $U_s(\cdot)$ be an increasing and strict concave utility function of the multicast session. Let the transmission cost on link $\langle i, J \rangle$ be $C_{iJ}(z_{iJ})$, where $C_{iJ}(\cdot)$ is an increasing strict convex function. According to network coding theory, to achieve a multicast rate r_s, it is necessary and sufficient to establish an information flow from s to each receiver $t \in T$, subject to the capacity constraints on all wireless links in the network. As illustrated in Figure 1, let x^t_{iJj} be the information flow for destination t on broadcast link $\langle i, J \rangle$ through relay j. Then the optimal multicast rate can be obtained by solving the following problem.

Baseline Capacity Model: Primal Problem

$$\max_{\{z_{iJ}\} \in \mathcal{Z}, \{x^t_{iJj}\}} U_s(r_s) - \sum_{\langle i,J \rangle \in H} C_{iJ}(z_{iJ}) \quad (1)$$

subject to:

$$\textstyle\sum_{j \in J} x^t_{iJj} \le z_{iJ}, \quad \forall \langle i, J \rangle \in H \quad \forall t \in T, \quad (2)$$

$$\textstyle\sum_{\langle i,J \rangle \in H, j \in J} x^t_{iJj} - \sum_{\langle m,I \rangle \in H | i \in I} x^t_{mIi} - r_s 1(i = s) \ge 0, \quad \forall t \in T, \quad \forall i \ne t, \quad (3)$$

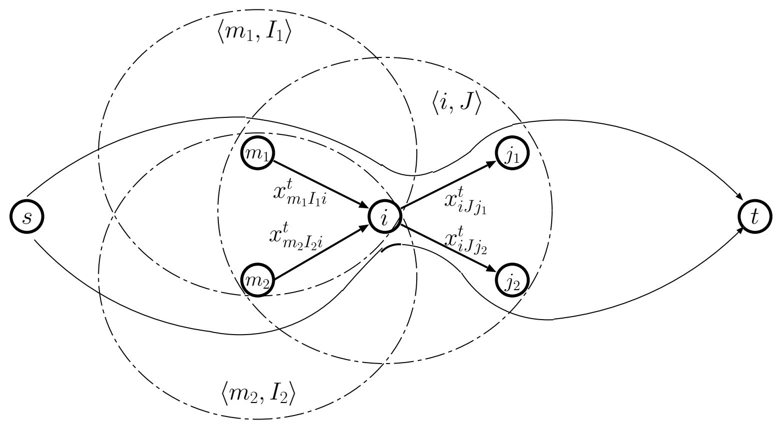

Figure 1: Multicast Information Flow over Multi-hop Wireless Network

where constraints (2) ensure the information flow through each broadcast link for each destination is bounded below the scheduled link rate, and constraints (3) represent the information flow conservation on each node for each destination. In the capacity model, r_s corresponds to source rate control, $\mathbf{z}$ corresponds to link scheduling, $\mathbf{x}$ corresponds to network coding based content scheduling and routing. In the model, if the source multicast rate is fixed, the cross-layer scheduling is optimized for minimum network-wide transmission cost. To account for battery capacity on individual nodes, one can increase the link cost $C_{iJ}(\cdot)$ as the battery life on node i decreases. This way, the scheduling solution will lead to more balanced battery consumption on all nodes.

When there is a set of sources S for the multicast session, rather than a single source, e.g., when the same content is available at multiple nodes, one can augment the network by adding a virtual super source $\bar{s}$ and virtual no-cost infinite-bandwidth links $\langle \bar{s}, s_i\rangle$, $\forall s_i \in S$. Multicast from the set S in the original network is equivalent to multicast from the single source $\bar{s}$ in the augmented network. When there are multiple multicast sessions in the network (unicast session can be treated as a single-source single-receiver multicast), *inter-session network coding* can potentially further improve the multicast efficiency on top of intra-session coding [1, 2]. However, the complexity of inter-session network coding is generally high. As will be discussed in Section 3, without considering inter-session coding, the capacity model is readily extended to multiple multicast/unicast sessions.

2 Optimal Cross-layer Scheduling

The joint optimization problem has a concave objective function, linear constraints and a convex feasible set for $\mathbf{z}$ and $\mathbf{x}$. It is a convex optimization problem, and can be solved by the corresponding dual problem without duality gap. One way to formulate the dual problem is to introduce lagrange multipliers to relax constraints (3). Let q_i^t be the lagrange multiplier for the information flow conservation constraint on node i for destination t (by default, we set $q_t^t = 0$). Then the Lagrangian can be formulated as

$$L(r_s, \mathbf{z}, \mathbf{x}, \mathbf{q}) \triangleq U(r_s) - \sum_{\langle i,J\rangle \in H} C_{iJ}(z_{iJ}) + \sum_{t\in T}\sum_{i\neq t} q_i^t \left(\sum_{\langle i,J\rangle\in H, j\in J} x^t_{iJj} - \sum_{\langle m,I\rangle \in H | i\in I} x^t_{mIi} - r_s 1(i=s)\right) \tag{4}$$

The dual function $D(\mathbf{q})$ is by definition the maximization of the Lagrangian

$$D(\mathbf{q}) \triangleq \max_{r_s, \mathbf{z}, \mathbf{x}} \quad L(r_s, \mathbf{z}, \mathbf{x}, \mathbf{q}) \tag{5}$$

$$\text{subject to: } \mathbf{z} \in \mathcal{Z}, \quad \sum_{j \in J} x^t_{iJj} \leq z_{iJ}, \quad \forall \langle i, J \rangle \in H, \quad \forall t \in T \tag{6}$$

Given multipliers $\mathbf{q}$, the dual function $D(\mathbf{q})$ can be obtained by solving two sub-problems.
Source Rate Control Sub-problem S_1:

$$S_1(\mathbf{q}) \triangleq \max_{r_s \geq 0} \quad U(r_s) - \sum_{t \in T} q^t_s r_s. \tag{7}$$

Link and Content Scheduling Sub-problem S_2:

$$S_2(\mathbf{q}) \triangleq \max_{\mathbf{z},\mathbf{x}} \sum_{t \in T} \sum_{i \neq t} q^t_i \left(\sum_{\langle i,J \rangle \in H, j \in J} x^t_{iJj} - \sum_{\langle m,I \rangle \in H | i \in I} x^t_{mIi} \right) - \sum_{\langle i,J \rangle \in H} C_{iJ}(z_{iJ}) \tag{8}$$

$$\text{subject to: } \mathbf{z} \in \mathcal{Z}, \quad \sum_{j \in J} x^t_{iJj} \leq z_{iJ}, \quad \forall \langle i, J \rangle \in H, \quad \forall t \in T \tag{9}$$

By changing the order of summation in (8), we have

$$\begin{aligned} S_2(\mathbf{q}) &\triangleq \max_{\mathbf{z} \in \mathcal{Z}, \sum_{j \in J} x^t_{iJj} \leq z_{iJ}} \sum_{t \in T} \sum_{\langle i,J \rangle \in H, j \in J} (q^t_i - q^t_j) x^t_{iJj} - \sum_{\langle i,J \rangle \in H} C_{iJ}(z_{iJ}) \\ &= \max_{\mathbf{z} \in \mathcal{Z}} \sum_{\langle i,J \rangle \in H} \left\{ \sum_{t \in T} \left(\max_{\sum_{j \in J} x^t_{iJj} \leq z_{iJ}} \sum_{j \in J} (q^t_i - q^t_j) x^t_{iJj} \right) - C_{iJ}(z_{iJ}) \right\} \\ &= \max_{\mathbf{z} \in \mathcal{Z}} \sum_{\langle i,J \rangle \in H} \left\{ \sum_{t \in T} [\max_{j \in J} (q^t_i - q^t_j)]^+ z_{iJ} - C_{iJ}(z_{iJ}) \right\} \\ &= \max_{\mathbf{z} \in \mathcal{Z}} \sum_{\langle i,J \rangle \in H} w_{iJ} z_{iJ} - C_{iJ}(z_{iJ}), \end{aligned}$$

where the weight of broadcast link $\langle i, J \rangle$ under multiplier $\mathbf{q}$ is

$$w_{iJ} \triangleq \sum_{t \in T} [\max_{j \in J} (q^t_i - q^t_j)]^+ \tag{10}$$

Then $D(\mathbf{q}) = S_1(\mathbf{q}) + S_2(\mathbf{q})$.

Due to the strong duality, we can obtain the primal optimum $\{\mathbf{z}^*, \mathbf{x}^*\}$ by solving the dual optimization $D(\mathbf{q}^*) = \min_{\mathbf{q} \geq 0} D(\mathbf{q})$. The dual optimization can be solved by the standard subgradient method. Specifically, $\mathbf{q}^*$ can be approached by the following iterative algorithm

$$\mathbf{q}(k+1) = [\mathbf{q}(k) - h(k)\xi(k)]^+, \tag{11}$$

where $\xi(k)$ is a subgradient of the dual function $D(\cdot)$ at $\mathbf{q}(k)$, $h(k)$ is the step size. Let $\{r_s(k), \mathbf{z}(k), \mathbf{x}(k)\}$ be the optimizing variables solving (7) and (8) at $\mathbf{q}(k)$. One subgradient can be obtained as

$$\xi^t_i(k) = \sum_{\langle i,J \rangle \in H, j \in J} x^t_{iJj}(k) - \sum_{\langle m,I \rangle \in H | i \in I} x^t_{mIi}(k) - r_s(k) 1(i = s) \tag{12}$$

To calculate $\xi(k)$, we have

$$\textbf{Source Rate Control:} \quad r_s(k) = \operatorname*{argmax}_{r_s} \quad U_s(r_s) - \sum_{t \in T} q^t_s r_s, \tag{13}$$

$$\textbf{Link Scheduling:} \quad \mathbf{z}(k) = \operatorname*{argmax}_{\mathbf{z} \in \mathcal{Z}} \sum_{\langle i,J \rangle \in H} w_{iJ} z_{iJ} - C_{iJ}(z_{iJ}), \tag{14}$$

Information Flow Routing: The information flow variables $x^t_{iJj}(k)$ can be calculated in the following ways:

- if $z_{iJ}(k) > 0$ and $\mathbf{max}_{j\in J}(q^t_i(k) - q^t_j(k)) > 0$
  1. let $b^t_i(k) \triangleq \mathbf{max}_{j\in J}(q^t_i(k) - q^t_j(k))$; and $B^t_i(k) = \{j|q^t_i(k) - q^t_j(k) = b^t_i(k)\}$
  2. $x^t_{iJj}(k) = 0$, if $j \notin B^t_i(k)$;
  3. $x^t_{iJj}(k) = \frac{z_{iJ}(k)}{|B^t_i(k)|}$, if $j \in B^t_i(k)$;
- otherwise, $x^t_{iJj}(k) = 0, \forall j, \forall t$;

Follow the convergence theorem of subgradient methods, if $\lim_{k\to\infty} h(k) = 0$ and $\sum_{k=1}^{\infty} h(k) = \infty$, the iterative algorithm converge to the dual optimum $D(\mathbf{q}^*)$, and the variables $\{r_s(k), \mathbf{z}(k), \mathbf{x}(k)\}$ converge to the primal optimum $\{r^*_s, \mathbf{z}^*, \mathbf{x}^*\}$.

2.1 Physical Interpretation

In equation (11), if we fix the step size to be $h(k) = h$, define $Q^t_i(k) = q^t_i(k)/h$, then we have

$$Q^t_i(k+1) = \left\{ Q^t_i(k) - \left[\sum_{\langle i,J\rangle\in H, j\in J} x^t_{iJj}(k) - \sum_{\langle m,I\rangle\in H|i\in I} x^t_{mIi}(k) - r_s(k)1(i=s) \right] \right\}^+ \tag{15}$$

Then $Q^t_i(k)$ is the the backlog of the *information flow queue* on node i for destination t. An information flow queue is the counterpart of the per-destination queue on each node in the optimal cross-layer scheduling for unicast flows [3]. $\mathbf{q}(k)$ coordinates the scheduling at three layers:

Source Rate Control – *Information Queue Backpressure*

In equation (13), the source rate is regulated by the derivative of the multicast utility function and the summation of the information queue back-pressure at the source over all its destinations. Recall for unicast, each source only reacts to back-pressure from its associated single destination.

Link Scheduling – *Maximum Information-Weight Matching (MIWM)*

In equation (14), the link weight for a broadcast link $\langle i, J\rangle$ is $w_{iJ} \triangleq \sum_{t\in T}[\mathbf{max}_{j\in J}(q^t_i - q^t_j)]^+$, i.e., the summation of the maximum differential queue length between node i and any neighbor in set J over all destinations. The link scheduling is calculated as the solution of the maximum information-weight matching problem defined in (14). MIWM is different from the Maximum Weight Matching (MWM) policy for the unicast case. In MWM, each link has one intended receiver, different flows to different destinations compete for bandwidth on a link, and the urgency of activating a unicast link is measured by the maximum differential queue length over all destinations. In multicast, one hyperlink can have multiple intended receivers. For a destination, the urgency of activating a hyper link is measured by the maximum differential queue length for the destination between the transmitter and its intended receivers. With perfect network coding, information flows to different destinations in the same multicast session have no bandwidth conflict on a hyperlink. They can be simultaneously carried by a hyperlink as long as their individual rates are smaller than the transmission rate of the hyperlink. In other words, activating a hyperlink can simultaneously benefit information flows to different destinations in the same session. Consequently, for a single multicast session, a hyperlink is weighted by the summation of the maximum differential queue lengths to all multicast destinations.

Routing – *Greedy Information Flow*

In unicast, when a link is activated, its link bandwidth is used to transmit the traffic for the destination maximizing the differential queue length. In multicast, when a hyperlink is activated, its link bandwidth is used to transmit information for all destinations with information backlog on the transmitter. If a broadcast link $\langle i, J\rangle$ is activated, for each destination, information only flows from node i to the subset of neighbors maximizing the differential queue length.

3 Multiple Multicast Sessions

When multiple multicast sessions overlap in time, they compete for the bandwidth available in the network. The content scheduling, routing and link activation of concurrent sessions are tightly coupled. Our baseline analysis can be extended to study the optimal sharing between them. Specifically, there is a set S of asynchronous P2P sharing sessions. For a session, let s be the source, (for a session with multiple sources, s is the virtual super-source), and T_s be the set of active receivers of source s. Without loss of generality, we assume there is no overlap between receiver sets of different sources. (If there is one node joins multiple multicast sessions, we can generate multiple virtual nodes, each of which joins one session, and is connected to the node with a infinite-bandwidth virtual link)

The optimal sharing between sessions can be formulated as:

$$\max_{\{z_{iJ}\}\in\mathcal{Z},\{x_{iJj}^{(s,t)}\}} \sum_{s\in S} U_s(r_s) - \sum_{\langle i,J\rangle\in H} C_{iJ}(z_{iJ}) \tag{16}$$

subject to

$$\textstyle\sum_{j\in J} x_{iJj}^{(s,t)} \le z_{iJ}^{(s)}, \quad \sum_{s\in S} z_{iJ}^{(s)} \le z_{iJ}, \quad \forall s\in S, \quad \forall t\in T_s, \quad \forall\langle i,J\rangle\in H \tag{17}$$

$$\textstyle\sum_{\langle i,J\rangle\in H, j\in J} x_{iJj}^{(s,t)} - \sum_{\langle m,I\rangle\in H | i\in I} x_{mIi}^{(s,t)} - r_s 1(i=s) \ge 0, \quad \forall s\in S, \quad \forall t\in T_s, \quad \forall i\ne t,, \tag{18}$$

where $x_{iJj}^{(s,t)}$ is the information flow from source s to its receiver t on broadcast link $\langle i, J\rangle$ through relay j, $z_{iJ}^{(s)}$ is the bandwidth share of session s on $\langle i, J\rangle$. Similar to the single-session case in Section 1, the optimal multi-session sharing can be obtained by a cross-layer scheduling policy.

Let q_i^t be the lagrange multiplier for the information flow conservation constraint (18) on node i for destination t (by default, we set $q_t^t = 0$). Then the Lagrangian can be formulated as

$$L^m(\mathbf{r},\mathbf{z},\mathbf{x},\mathbf{q}) \triangleq \sum_{s\in S} U_s(r_s) - \sum_{\langle i,J\rangle\in H} C_{iJ}(z_{iJ}) + \sum_{t\in T}\sum_{i\ne t} q_i^t \left(\sum_{\langle i,J\rangle\in H, j\in J} x_{iJj}^{(s,t)} - \sum_{\langle m,I\rangle\in H|i\in I} x_{mIi}^{(s,t)} - r_s 1(i=s)\right) \tag{19}$$

The dual function $D^m(\mathbf{q})$ is by definition the maximization of the Lagrangian

$$D^m(\mathbf{q}) \triangleq \max_{\mathbf{r},\mathbf{z},\mathbf{x}} \quad L^m(\mathbf{r},\mathbf{z},\mathbf{x},\mathbf{q}) \tag{20}$$

$$\text{subject to: } \sum_{j\in J} x_{iJj}^{(s,t)} \le z_{iJ}^{(s)}, \quad \sum_{s\in S} z_{iJ}^{(s)} \le z_{iJ}, \quad \mathbf{z}\in\mathcal{Z}, \quad \forall s\in S, \quad \forall t\in T_s, \quad \forall\langle i,J\rangle\in H \tag{21}$$

Given multipliers $\mathbf{q}$, the dual function $D(\mathbf{q})$ can be obtained by solving two sub-problems.
Source Rate Control Sub-problem S_1^m:

$$S_1^m(\mathbf{q}) \triangleq \max_{r_s\ge 0} \quad U_s(r_s) - \sum_{t\in T} q_s^t r_s. \tag{22}$$

Link and Content Scheduling Sub-problem S_2^m:

$$S_2^m(\mathbf{q}) \triangleq \max_{\mathbf{z},\mathbf{x}} \sum_{s\in S}\sum_{t\in T_s}\sum_{i\ne t} q_i^t \left(\sum_{\langle i,J\rangle\in H, j\in J} x_{iJj}^{(s,t)} - \sum_{\langle m,I\rangle\in H|i\in I} x_{mIi}^{(s,t)}\right) - \sum_{\langle i,J\rangle\in H} C_{iJ}(z_{iJ}) \tag{23}$$

$$\text{subject to: } \sum_{j\in J} x_{iJj}^{(s,t)} \le z_{iJ}^{(s)}, \quad \sum_{s\in S} z_{iJ}^{(s)} \le z_{iJ}, \quad \mathbf{z}\in\mathcal{Z}, \quad \forall s\in S, \quad \forall t\in T_s, \quad \forall\langle i,J\rangle\in H \tag{24}$$

By changing the order of summation in (23), we have

$$\begin{aligned}
S_2^m(\mathbf{q}) &\triangleq \max_{\sum_{j\in J} x_{iJj}^{(s,t)}\le z_{iJ}^{(s)}, \sum_{s\in S} z_{iJ}^{(s)}\le z_{iJ}, \mathbf{z}\in\mathcal{Z}} \sum_{s\in S}\sum_{t\in T_s}\sum_{\langle i,J\rangle\in H, j\in J} (q_i^t - q_j^t)x_{iJj}^{(s,t)} - \sum_{\langle i,J\rangle\in H} C_{iJ}(z_{iJ}) \\
&= \max_{\mathbf{z}\in\mathcal{Z}} \sum_{\langle i,J\rangle\in H} \left\{ \max_{\sum_{s\in S} z_{iJ}^{(s)}\le z_{iJ}} \sum_{s\in S} \max_{\sum_{j\in J} x_{iJj}^{(s,t)}\le z_{iJ}^{(s)}} \sum_{t\in T_s} \left(\sum_{j\in J} (q_i^t - q_j^t) x_{iJj}^{(s,t)} \right) - C_{iJ}(z_{iJ}) \right\} \\
&= \max_{\mathbf{z}\in\mathcal{Z}} \sum_{\langle i,J\rangle\in H} \left\{ \max_{\sum_{s\in S} z_{iJ}^{(s)}\le z_{iJ}} \sum_{s\in S}\sum_{t\in T_s} \left([\max_{j\in J}(q_i^t - q_j^t)]^+ z_{iJ}^{(s)} \right) - C_{iJ}(z_{iJ}) \right\} \\
&= \max_{\mathbf{z}\in\mathcal{Z}} \sum_{\langle i,J\rangle\in H} \left\{ \max_{\sum_{s\in S} z_{iJ}^{(s)}\le z_{iJ}} \sum_{s\in S} w_{iJ}^{(s)} z_{iJ}^{(s)} - C_{iJ}(z_{iJ}) \right\} \\
&= \max_{\mathbf{z}\in\mathcal{Z}} \sum_{\langle i,J\rangle\in H} w_{iJ} z_{iJ} - C_{iJ}(z_{iJ}),
\end{aligned}$$

where the weight of broadcast link $\langle i, J\rangle$ under multiplier $\mathbf{q}$ is

$$w_{iJ} \triangleq \max_{s\in S} w_{iJ}^{(s)}, \quad \text{with} \quad w_{iJ}^{(s)} \triangleq \sum_{t\in T_s} [\max_{j\in J}(q_i^t - q_j^t)]^+ \tag{25}$$

Then $D^m(\mathbf{q}) = S_1^m(\mathbf{q}) + S_2^m(\mathbf{q})$.

Due to the strong duality, we can obtain the primal optimum $\{\mathbf{z}^*, \mathbf{x}^*\}$ by solving the dual optimization $D^m(\mathbf{q}^*) = \min_{\mathbf{q}\ge 0} D^m(\mathbf{q})$. The dual optimization can be solved by the standard subgradient method. Specifically, $\mathbf{q}^*$ can be approached by the following iterative algorithm

$$\mathbf{q}(k+1) = [\mathbf{q}(k) - h(k)\xi(k)]^+, \tag{26}$$

where $\xi(k)$ is a subgradient of the dual function $D^m(\cdot)$ at $\mathbf{q}(k)$, $h(k)$ is the step size. Let $\{\mathbf{r}(k), \mathbf{z}(k), \mathbf{x}(k)\}$ be the optimizing variables solving (22) and (23) at $\mathbf{q}(k)$. One subgradient can be obtained as

$$\xi_i^t(k) = \sum_{\langle i,J\rangle\in H, j\in J} x_{iJj}^{(s,t)}(k) - \sum_{\langle m,I\rangle\in H | i\in I} x_{mIi}^{(s,t)}(k) - r_s(k)1(i=s) \tag{27}$$

To calculate $\xi(k)$, we have

$$\textbf{Source Rate Control:} \qquad r_s(k) = \operatorname*{\mathbf{argmax}}_{r_s} \; U_s(r_s) - \sum_{t\in T} q_s^t r_s, \tag{28}$$

$$\textbf{Link Scheduling:} \qquad \mathbf{z}(k) = \operatorname*{\mathbf{argmax}}_{\mathbf{z}\in\mathcal{Z}} \sum_{\langle i,J\rangle\in H} w_{iJ} z_{iJ} - C_{iJ}(z_{iJ}), \tag{29}$$

Information Flow Routing: The information flow variables $x_{iJj}^{(s,t)}(k)$ can be calculated in the following ways:

- if $z_{iJ}(k) > 0$, let $S_{iJ}(k) = \{s | w_{iJ}^{(s)}(k) = w_{iJ}(k)\}$, then $z_{iJ}^{(s)} = 0$, if $s \notin S_{iJ}(k)$; $z_{iJ}^{(s)} = \frac{z_{iJ}(k)}{|S_{iJ}(k)|}$, if $s \in S_{iJ}(k)$, for $\forall t \in T_s$, let $b_i^t(k) \triangleq \mathbf{max}_{j\in J}(q_i^t(k) - q_j^t(k))$, if $b_i^t(k) > 0$, then

  1. let $B_i^t(k) = \{j | q_i^t(k) - q_j^t(k) = b_i^t(k)\}$

2. $x_{iJj}^{(s,t)}(k) = 0$, if $j \notin B_i^t(k)$;
3. $x_{iJj}^{(s,t)}(k) = \frac{z_{iJ}^{(s)}(k)}{|B_i^t(k)|}$, if $j \in B_i^t(k)$;

- Otherwise, $x_{iJj}^{(s,t)}(k) = 0, \forall j, \forall s, \forall t$;

With multiple multicast sessions, when a broadcast link $\langle i, J \rangle$ is activated, its link bandwidth is equally shared between all sessions maximizing the information link weight $w_{iJ}^{(s)}$. Within each session, similar to the single multicast session case, for each destination, information only flows from node i to the subset of neighbors maximizing the differential queue length.

Follow the convergence theorem of subgradient methods, if $\lim_{k\to\infty} h(k) = 0$ and $\sum_{k=1}^{\infty} h(k) = \infty$, the iterative algorithm converge to the dual optimum $D^m(\mathbf{q}^*)$, and the variables $\{\mathbf{r}(k), \mathbf{z}(k), \mathbf{x}(k)\}$ converge to the primal optimum $\{\mathbf{r}^*, \mathbf{z}^*, \mathbf{x}^*\}$.

The current analysis assumes there is no content coding cross different sessions. As demonstrated in [2], inter-session network coding can provide additional gain. We will investigate the adoption of inter-session coding in future our analysis and designs.

References

[1] AHLSWEDE, R., CAI, N., LI, S.-Y. R., AND YEUNG, R. W. Network information flow. *IEEE Trans. on Information Theory 46* (2000), 1204–1216.

[2] KATTI, S., RAHUL, H., HU, W., KATABI, D., MEDARD, M., AND CROWCROFT, J. XORs In The Air: Practical Wireless Network Coding. In *Proceedings of ACM SIGCOMM* (2006).

[3] TASSIULAS, L., AND EPHREMIDES, A. Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks. *IEEE Transactions on Automatic Control 37*, 12 (December 1992), 1936–1949.